A theory of strategic venture investing

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Abstract

Some venture capital investors seek purely financial gains while others, such as corporations, also pursue strategic objectives. The paper examines a model where a strategic investor can achieve synergies, but can also face a conflict of interest with the entrepreneur. If the start-up is a complement to the strategic partner, it is optimal to obtain funding from the strategic investor. If the start-up is a mild substitute, the entrepreneur prefers an independent venture capitalist. With a strong substitute, syndication becomes optimal, such that the independent venture capitalist is the active lead investor and the strategic partner a passive co-investor. The expected returns for the entrepreneur are nonmonotonic, lowest for a mild substitute, and higher for a strong substitute as well as for a complement. The paper also explains why a strategic investor often pays a higher valuation than an independent venture capitalist. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Some investors choose to make investments in entrepreneurial ventures not merely for financial returns, but also for strategic reasons. Intel, for example, invested over
$1 billion in 1999 in over 250 companies, stating that its “goals were much more strategic than financial”. How is a strategic investor like Intel different from a purely financial investor, such as an independent venture capitalist? What does it mean to be a strategic investor? And how do entrepreneurs view an investor with a strategic objective?

With the growing importance of entrepreneurial ventures, these questions are becoming increasingly pertinent. Not only is the venture capital industry growing dramatically, but it is also populated by an increasingly diverse set of investors. In addition to the traditional purely financial investors (namely the many private independent venture capital partnerships), corporations as diverse as Microsoft, UPS, or Reuters hope to leverage their existing businesses into the venture capital market. There are investment banks (such as Hambrecht and Quist) that hope to extend their expertise and relationships into venture investing. There are some network-based concepts of strategic venture investing, such as the Kleiner Perkins Java fund, which combines the interests of a number of “Java-centric” corporations, or CMGI which is an Internet-focused business incubator. Most recently even the Central Intelligence Agency (CIA) decided to launch its own strategic venture capital fund.

What objectives do these so-called “captive” venture capitalists pursue? Consider the following press release, announcing a $1 billion fund by CMGI and its venture capital affiliate, @Ventures (January 24th, 2000):

The new venture fund will focus exclusively on investing in and supporting the rapid development and growth of Internet enabling technologies and infrastructure companies that are synergistic with the CMGI and @Ventures network.

Achieving synergies with their core business is the leading explanation for these strategic venture investments. In a survey of corporate venture capitalists, Yost and Devlin (1993) find that 93% of all respondents considered strategic returns a main objective. Achieving synergies is typically viewed as something desirable. But there is a question of exactly who benefits from them? In particular, it is not obvious that entrepreneurs would actually want their investors to be pursuing synergies. For example, in an extensive study of corporate venture investing, Block and MacMillan (1993) note that entrepreneurs can distrust corporate investors because “they will control their ventures to satisfy corporate objectives at the expense of the ventures’ well being”.

This paper provides a theoretical foundation to address the topic of strategic venture investments. It examines a model in which a strategic investor competes with independent venture capitalists to finance an entrepreneur’s new venture. An important aspect of venture financing is the support provided by the investor.\footnote{A growing literature discusses the role of venture capital investors supporting the development of their start-up companies. See in particular Gompers (1995, 1997), Hellmann (1998), Hellmann and Puri (2000, 2001), Lerner (1995a), and Sahlman (1990). The idea that the identity of the investor matters is also important in the banking literature. See, in particular, Diamond (1991) and Rajan (1992).} Investors compete not only on the price of equity (i.e., the valuation), but also with...
the level of support that they can credibly provide. We define a strategic investor as an investor that owns some asset whose value is affected by the new venture. From this definition we derive that an independent venture capitalist only pursues financial objectives, while the strategic investor also cares about the new venture’s strategic impact. The model allows for both positive and negative interactions, i.e., success of the new venture could complement or cannibalize the strategic investor’s asset.

Depending on the underlying characteristics of the new venture, our analysis uncovers three distinct optimal financing arrangements. If the new venture is a complement, the entrepreneur chooses the strategic investor. Although the strategic investor’s level of support is below the first-best level, it is higher than what a venture capitalist would provide. But if the venture is a substitute, then the entrepreneur prefers venture capital financing. Even though the entrepreneur could extract a higher valuation out of the strategic investor, the independent venture capitalist offers more support ex post. This result holds when the strategic investor is equally or less able than a venture capitalist, and it frequently continues to hold even when the strategic investor is more able. Moreover, it only holds when the venture poses a relatively small threat to the strategic investor’s asset. A third type of financing arises when the new venture poses a large threat. In this case, there is syndicated financing. The venture capitalist is the lead investor, actively supporting the venture and typically holding a board seat. The strategic investor, by contrast, remains a passive investor, not providing support nor holding a board seat. In fact, the strategic investor’s main role is to hold equity, so as to reduce the venture capitalist’s stake in the new venture. This reduces the venture capitalist’s support for the new venture and thereby curbs excessive cannibalization.

The model also makes predictions about the valuation that the entrepreneur can obtain from the two types of investors. We find that for a wide range of parameters the strategic investor is willing to pay a higher valuation, i.e., take a smaller equity stake. However, we also show that the so-called post-money valuation, which is the standard way of valuing firms in the venture capital industry, can be a very misleading indicator of underlying firm values. This is because it does not reflect the intangible value of investor support. If we consider an accurate measure of value, such as the expected return for the entrepreneur, then we find another interesting result. The more a new venture threatens to cannibalize the value of existing assets, the lower its chances of success, but the higher its value for the entrepreneur.

We also consider a number of extensions of the model that show the robustness of the results and provide some additional insights. For example, we show that if the strategic investor and the venture capitalist have very distinct abilities, then it is possible to have syndicated finance where both types of investors hold board seats and actively provide support. Moreover, if there is more than one strategic investor, then it is also possible that several of them will hold passive stakes.

This work is related to a number of papers in the financial contracting literature. Aghion and Tirole (1994), for example, derive conditions under which entrepreneurs might seek external finance to alleviate a hold-up problem with a contracting partner. The question of how ownership of one asset affects the ability to invest in other assets is also central to the analysis of conglomerate structures (see, e.g.,
Gertner et al., 1994; Stein, 1997). The paper is also related to the work by Anton and Yao (1994, 1995), Anand and Galetovic (2000), and Gans and Stern (2000) on contracting between entrepreneurs and established corporations in the presence of weak intellectual property rights.\(^2\) It also contributes to the growing literature on finance and product market interactions (see, e.g., Brander and Lewis, 1986; Maksimovic and Titman, 1991). While much of this literature has been concerned with how financing affect a firm’s strategy, this paper examines how the strategy pursued by the financiers affects firms. On the empirical side, Gompers and Lerner (2000) provide some evidence on corporate venture capital. They find that the main measurable difference between corporate and independent venture capitalists is that corporations pay higher valuations. This is consistent with the analysis of this paper.\(^3\)

The remainder of the paper is organized as follows. Section 2 introduces the basic model. Section 3 derives the optimal financing pattern. Section 4 explores the difference between valuation and value. Section 5 examines a variety of model extensions. Section 6 concludes. All formal proofs are relegated to Appendix A.

2. The model

Consider a risk-neutral world with no discounting. An entrepreneur (E) wants to start a new venture, but has no wealth. She requires an amount \(I\) to fund the venture. She can approach a competitive pool of independent venture capitalists (V) and/or a strategic investor (S). There are two possible future states of nature that we call success and failure. Success occurs with probability \(q\).

Consider first the case in which \(E\) contracts with \(V\). If the venture fails there are no returns, and we normalize the ex post utilities by \(u_E = 0\) and \(u_V = 0\). In case of success, the new venture generates transferable profits that are normalized to one, and a nontransferable value of \(\beta \geq 0\) to \(E\).\(^4\) Let \(\alpha_V \in [0, 1]\) be \(V\)’s share in the profits, so that \(u_V = \alpha_V\) and \(u_E = 1 + \beta - \alpha_V\) in case of success. This simple specification

\(^2\)While lack of intellectual property protection might be a generic problem, this literature relies on a stronger assumption, namely that contracting increases the leakage of intellectual property to the entrepreneur’s disadvantage. The current paper abstracts from intellectual property concerns, although it is straightforward to see how these would only add to the problems of strategic investing derived in this paper.

\(^3\)There is also a large business literature that discusses the advantages and disadvantages of corporate venture capital. Some useful references include Hardymon et al. (1983), Kanter et al. (1990), Siegel et al. (1988), Sykes and Block (1989), and Winters and Murfin (1988). For some illustrative case studies, see Conneely and Gompers (1999), Hellmann et al. (1995), or Lerner (1995b). For some useful journalistic accounts, see San Francisco Chronicle (1997), Wall Street Journal (1998, 2000), or Red Herring (2000).

\(^4\)All the results of the paper go through with \(\beta = 0\). However, \(\beta > 0\) may be more realistic for at least three reasons. First, consistent with much of the finance literature, \(E\) may have the ability to enjoy some on-the-job consumption in the event that the company is successful. Second, the success of the new venture is likely to increase the market’s perception of \(E\)’s human capital and \(\beta\) may represent the increased rents from that. Finally, it may simply be that \(E\) derives utility directly from the success of the new venture, be it personal satisfaction, or the enjoyment of creating value.
eliminates any considerations of capital structure from the analysis. Without loss of
generality we can represent ownership stakes with equity stakes.5

\( V \) is a purely financial investor that only cares about the profits generated by
the new venture. \( S \) differs from \( V \) in one important dimension: \( S \) owns an asset that is
affected by the performance of the new venture. The asset can be thought of in a
number of ways, most naturally as a line of business, but also as physical assets,
contractual assets, or even human capital. If \( E \) contracts with \( S \) instead of \( V \), \( E \) will
also get \( u_E = 0 \) in case of failure and \( u_E = 1 + \beta - \alpha_S \) in case of success. The value of
\( S \)'s asset is given by \( u_S \) in case of failure and \( \bar{u}_S \) in case of success, so that \( S \)'s ex post
utility is given by \( u_S = \gamma_S \) in case of failure and \( u_S = \bar{u}_S + \alpha_S \) in case of success. \( u_S \)
and \( \bar{u}_S \) can be thought of as the reduced form utilities of the potentially very
complicated interaction between the new venture and the strategic investor.6

We define \( \theta = \bar{u}_S - u_S \), then \( \theta \) measures the change in the value of the strategic
investor’s asset, if the new venture were to succeed. \( \theta \) measures the net impact of all
the strategic actions between the new venture and assets of the strategic investor. It
summarizes how valuable the success of the new venture would be for \( S \)'s existing
asset. If \( \theta > 0 \) we say that \( S \) has a complementary asset, and if \( \theta < 0 \) the new venture
partially substitutes (or cannibalizes) \( S \)'s asset. Most models of competition suggest
\( \theta < 0 \), as the industry changes from an oligopoly of \( n \) firms to one with \( n + 1 \) firms.
\( \theta > 0 \) may arise because of demand externalities, such as between software and
hardware (Katz and Shapiro, 1994) or because of cost complementarities (such as
through the reduction of input prices). \( \theta \) also corresponds directly to well-known
concepts in strategic management, namely competence-enhancing and competence-
destroying innovations (Tushman and Anderson, 1986). Finally, the model is
sufficiently general to accommodate an interpretation where managers do not always
act in the best interest of the shareholders (Jensen and Meckling, 1976). In this case,
\( \theta \) represents the impact on the incumbent management team’s utility, which may
differ from the shareholder’s utility functions.7

Whether the new venture succeeds or fails depends on the support of the investor.
In the context of venture capital investments, investors can engage in a number of
activities that increase the value of the venture. These activities are by and large
subtle so that they cannot be contracted upon. Even if some actions are contractible,
what matters is that there are some others that are not contractible. The level of
support is then endogenously determined by the investor’s incentives. We model this

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5 A growing literature addresses the question of security design in venture capital. Admati and Pfleiderer
(1999), or Trester (1998) discuss this question. Gompers (1997) and Kaplan and Strömberg (1999) also
provide some empirical evidence.

6 Since we only care about the expected utility levels, we do not need to model these interactions. They
potentially include competition, externalities across markets, licensing, acquisitions, and others. The utility
levels also include the fact that if the venture does not succeed, other ventures could succeed at a later time,
or with a slightly different approach.

7 We can think of an independent venture capitalist as an investor with \( \theta = 0 \). And if a venture capitalist
also has some other asset that may be affected by the new venture (such as an existing portfolio
investment) then we can think of that venture capitalist as an \( S \) rather than a \( V \).
with a simple linear quadratic model. Let \( q_i = P_i + p_i \sigma_i \), where \( \sigma_i \) is the level of support provided by \( i = V, S \), at a private cost \( c(\sigma_i) = \frac{1}{2} \sigma_i^2 \).

In addition to having a strategic objective, we also allow \( S \) to differ from \( V \) in a number of other dimensions. In particular, the base probability of success can differ for \( S \) and \( V \), as captured by \( P_S \) and \( P_V \). And the extent to which their support further increases the base probability of success can also differ for \( S \) and \( V \), as captured by \( p_S \) and \( p_V \). In general there can be a large number of investor- and deal-specific factors that influence these ability parameters, such as experience, technology, or fit. The model allows for this heterogeneity, and we say that \( S \) is more (less) able than \( V \) if \( P_S > (\ <) P_V \) and/or \( p_S > (\ <) p_V \).

We assume that investors can only duplicate each other’s support activities (we relax this assumption in Section 5.3). As a consequence, only one investor actively provides support. The other investor, however, can still hold a passive stake. The active investor can hold a board seat and/or other control rights, whereas the passive investor has no board seats or other control rights.

If \( V \) is the active investor, we denote the ex ante utilities of all parties by

\[
U^V_E = q_V(1 + \beta - \alpha_V - \alpha_S), \quad (1)
\]

\[
U^V_V = q_V \alpha_V - c(\sigma_V) - I_V, \quad (2)
\]

\[
U^V_S = u_S + q_V(\alpha_S + \theta) - I_S, \quad (3)
\]

where \( q_V = P_V + p_V \sigma_V \) and \( I_S + I_V = I \). For \( I_S = \alpha_S = 0 \), we talk of pure \( V \)-financing, whereas for \( I_S > 0 \) and \( \alpha_S > 0 \), we talk of mixed financing, where \( S \) is a passive investor.

If \( S \) is the active investor then \( V \) never becomes a passive investor (see Appendix A). We can therefore limit ourselves to pure \( S \)-financing. We have

\[
U^S_E = q_S(1 + \beta - \alpha_S), \quad (4)
\]

\[
U^S_S = u_S + q_S(\alpha_S + \theta) - c(\sigma_S) - I, \quad (5)
\]

where \( q_S = P_S + p_S \sigma_S \).

We make some assumptions on parameter values. First, we assume that \( 1 + \beta + \theta > 0 \), so that the value of the new venture in case of success is always positive. Second, we assume that \( I < P_V + \frac{1}{2} p_V^2 \). This guarantees that the project can always be financed by a competitive \( V \). Third, we assume that \( P_V > (1 + \beta) p_V^2 \) and \( P_S > (1 + \beta - \theta) p_S^2 \). This guarantees that the utility frontier between \( E \) and \( V \) or \( S \) is always downward sloping.

We assume that \( 0 \leq I_j \leq I, \ j = S, V \), and that no other transfer payments are possible at date zero between the three parties. This assumption is standard in a moral hazard model and ensures that the agent, in this case the investor, cannot simply buy the asset. It can be justified by a simple no-free-profits assumption that says that no imposter can ever make a profit by pretending to be one of the involved parties. More specifically, assume that there is a large pool of impostors with zero reservation utility. Whenever there are positive transfer payments, they can mimic the receiving party and take the money. As a consequence, no party will ever make
any upfront payments. Moreover, we assume that \( \alpha_i \in [0, 1] \), which also follows from the no-free-profits assumption. If \( \alpha_i < 0 \) were possible for some \( i \), then the other two parties would have an incentive to claim the venture was successful, even when it was not, and thereby make free profits. Apart from this, all parties have symmetric information.

We also need to specify how contracts are generated. As a base case we use a simple game in which \( E \) has all the bargaining power. The contract game is as follows. \( E \) can make a take-it-or-leave-it offer to \( S \), who can either accept or reject the offer. \( E \)'s take-it-or-leave-it offer to \( S \) can also stipulate syndication, in which case both \( S \) and some designated \( V \) have to accept the offer. If \( E \) does not want to make an offer to \( S \), or if the offer was rejected, \( E \) can access a perfectly competitive market of \( V \)'s. To model the competitive market of \( V \)'s, we assume that \( E \) can make a take-it-or-leave-it offer to any \( V \) (we would obtain exactly the same outcome if the \( V \)'s make competitive offers to \( E \)). In Section 5.1 we will examine the importance of this game structure. Contracts are written at date zero. Between date zero and one the active investor provides noncontractible support. At date one the project is either successful or not, and returns are paid out.

3. Optimal choice of investors

To solve for the optimal contract, we will first consider what happens if no deal is made with \( S \). In this case \( E \) approaches the competitive pool of \( V \), and obtains a contract \( q_V \), so that \( U^E_V(q_V) = 0 \equiv U_V \). \( S \)'s utility is by \( U^S_V(q_V) = u_S + q_V \theta \equiv U_S \). Consider now \( E \)'s optimal offer to \( S \). If she wants \( S \) to be the active investor, she maximizes \( U^E_S \), subject to \( U^S_S \geq U_S \). Naturally, we also have \( U^S_V \geq U_V \). If she wants \( V \) as the active investor, she maximizes \( U^E_V \), subject to \( U^S_V \geq U_S \) and \( U^V_V \geq U_V \). Note that the case of pure \( V \)-financing is a special case of this, in which \( E \) does not actually have to make an offer to \( S \). With downwardsloping utility frontiers, these participation constraints are always binding, so that \( U^i_j = U^i_j \) for all \( i = S, V \) and \( j = S, V \). Since \( U^S_S \) and \( U^V_V \) do not depend on the offer to \( S \), maximizing \( U^i_j \) is equivalent to maximizing \( U^E_S + U^S_S + U^V_V \), \( i = S, V \). But this is also equivalent to maximizing the sum of utilities \( W_i = U^i_E + U^i_S + U^i_V \). This is a useful transformation. It allows us to express \( E \)'s optimal offer as the contract that maximizes \( W = \text{Max}[W_S, W_V] \).

The choice of investor depends on the level of support provided by \( S \) and \( V \). This depends on the endogenously determined equity stakes \( \alpha_i \), \( i = S, V \). Consider the privately optimal choices from the first-order conditions:

\[
\sigma_S = p_S(\alpha_S + \theta), \quad (6)
\]

\[
\sigma_V = p_V \alpha_V. \quad (7)
\]

By contrast, the efficient level of support, that would maximize \( W_i \) given by

\[
\sigma^*_i = p_i(1 + \beta + \theta). \quad (8)
\]
There are several reasons why an investor provides an inefficient level of support. First, the investor only considers his share of profits, $\alpha$, ignoring the profits that accrue to $E$, $(1-\alpha)$. Second, the investor does not take into account $E$’s private benefits $\beta$. Third, if $V$ is the active investor, he does not take into account the externality $\theta$ on $S$’s other asset. And finally it can be that the less efficient investor is chosen.

Without the no-free-profits constraint, it would be easy to achieve the efficient equilibrium. For $W_S(\sigma_S^*) > W_V(\sigma_V^*)$, $S$ would be the active investor, and $\alpha_S = 1 + \beta$. $\sigma_S$ would then maximize $W_S$, and transfer payments would be used to ensure $U_V = U_1$ and $U_S = U_0$. Similarly, for $W_S(\sigma_S^*) < W_V(\sigma_V^*)$, $V$ would be the active investor, and $\alpha_V = 1 + \beta + \theta$ would achieve the efficient outcome. These arrangements, however, are rather artificial. Consider $W_S(\sigma_S^*) > W_V(\sigma_V^*)$: $S$ would not only have to make a transfer payment to $E$ to buy all the returns of the new venture (i.e., buy out the venture), but it would also have to make a transfer payment to $V$, so that in return $V$ offers an additional incentive payment $\beta$ to $S$. Such transfer payments are rarely observed in reality, partly because parties do not trust each other enough to transfer large sums of money for financing a new and unproven opportunity. We capture this with our assumption of no-free-profits, which eliminates these artificial solutions. It allows us to focus on outcome-based equity contracts that resemble much more closely the contracts actually used.

Consider thus the model with the no-free-profits constraint. Define $\theta_0$ from $\sigma_V(\theta_0) = \sigma_V^*$ and $\theta_1$ from $U^S_E(\theta_1) = U^V_E(\theta_1)$.

**Proposition 1 (Equilibrium financing pattern).** Suppose $S$ and $V$ are equally able.

(i) If $\theta > \theta_1 = 0$, then there is pure $S$-financing. Relative to the first-best, there is too little support, i.e., $\sigma_S < \sigma_V^*$.

(ii) If $0 < \theta < \theta_1 = 0$, then there is pure $V$-financing. Relative to the first-best, there is too little support, i.e., $\sigma_V < \sigma_V^*$.

(iii) If $\theta < \theta_0$, then there is mixed financing, where $V$ is the active and $S$ is the passive investor. The level of support is first-best, i.e., $\sigma_V = \sigma_V^*$.

Proposition 1 shows that even though $S$ and $V$ may be equally able investors, the choice of investor still matters. If the new venture is a complement to $S$, then $S$ is the optimal investor. But if the new venture is a substitute to $S$’s existing asset, then $E$ prefers to have $V$ as active investor. Moreover, if the new venture is only a mild threat, $V$ will be the sole investor, whereas if it is a strong threat, there will then be a syndicate in which $V$ is the active and $S$ the passive investor.

To provide some intuition for this result, consider Fig. 1. It exploits the property that $E$’s preferred solution is also the one that maximizes $W$. It shows the endogenous levels of $q_S$ and $q_V$. These depend on the support $\sigma_i$, which are a function of $\theta$ and $x_i$, where $x_i$ is chosen endogenously to satisfy $U_i = U_i$, $i = S, V$. In Appendix A we show that $q_V(\theta)$ is constant, $q_S(\theta)$ is increasing in $\theta$, and $q_S(\theta)$ equals $q_V(\theta)$ at $\theta = 0$. Moreover, $q_S^*(\theta)$ always lies above $q_S(\theta)$.

Consider now $\theta > 0$. Both $S$ and $V$ provide too little support relative to the first-best. But $S$ always provides a higher level of support than $V$. Hence $S$-financing is
more efficient. This explains the first part of Proposition 1. The situation changes for \( \theta < 0 \). \( V \) now provides the higher level of support than \( S \). For \( \theta \) not too negative, pure \( V \)-financing now dominates. The reason that \( S \) loses out as an investor is that it cannot commit to provide an adequate level of support. This explains the second part of Proposition 1. As \( \theta \) becomes strongly negative, however, yet another situation arises. For \( \theta < \theta_0 \), we have \( \sigma^*(\theta) < \sigma_V(\theta) \), i.e., the efficient level of support drops below that offered by \( V \). \( E \) can find a better solution by asking \( S \) to co-finance the project. \( S \) is eager to take some of the equity, to reduce \( V \)'s incentives. The outcome is syndicated financing, in which \( V \) remains the active investor, but \( S \) invests as a passive investor without control.

**Proposition 2** (Comparative statics).

(i) An increase in \( \beta \) decreases \( \theta_0 \) and leaves \( \theta_1 \) constant.
(ii) An increase in \( I \) increases \( \theta_0 \) and leaves \( \theta_1 \) constant.
(iii) An increase in \( P_S \) and/or \( p_S \) leaves \( \theta_0 \) constant and decreases \( \theta_1 \).
(iv) An increase in \( P_V \) and/or \( p_V \) decreases \( \theta_0 \) and increases \( \theta_1 \).

Proposition 2 shows how the three different financing regions depend on model parameters. Fig. 2 shows that an increase in \( \beta \) will not affect region in which
$S$-financing is optimal. However, it will increase the range in which $V$-financing is optimal and reduce the range in which mixed financing is optimal. The intuition is that if $E$ cares more about success per se, she is less interested in having $S$ absorb equity to reduce $V$’s support. From Fig. 3 we see that a decrease in $I$ will have the same effect as an increase in $\beta$. The reason here, however, is that the lower $I$ implies a lower $\theta_1$, and thus lower support $\sigma_V$ by $V$. $V$’s thus overinvest less often, and hence there is less mixed finance. Figs. 2c and d consider the case in which $S$ and $V$ differ in terms of underlying ability. Fig. 4 considers the case in which $S$ becomes more able than $V$, i.e., $P_S > P_V$ and/or $p_S > p_V$. The range in which $S$-financing is optimal becomes larger, and now also includes some cases in which $\theta < 0$. This is because a higher ability now compensates for $S$’s strategic motivation. The range in which $V$-financing is optimal naturally shrinks. The range of mixed financing, however, is not affected, as $V$ is the active investor under both $V$-financing and mixed financing. Finally, consider Fig. 5 where $V$ becomes a more able investor than $S$, i.e., $P_V > P_S$ and/or $p_V > p_S$. In this case, the range of $V$-financing increases, whereas both $S$-financing and mixed financing become less likely.

The key insight from this section is that with non-contractible support the choice of investor matters. Even if $S$ is more able than $V$, the opportunity to internalize externalities actually becomes a competitive disadvantage. In particular, it prevents $S$ from providing an adequate level of support to $E$. Ex ante $S$ would like to commit...
Fig. 3. An increase of $I$ from $I_1$ to $I_2$ will not affect the efficient level $q^*$, but increase $S$’s and $V$’s effort levels $q_S$ and $q_V$. This leads to an increase in the range where mixed financing is optimal.

Fig. 4. An increase of $P_S$ from $P_S^1$ to $P_S^2$ increases the efficient effort level under $S$-financing, but not under $V$-financing. It also increases the optimal effort for $S$ but not for $V$. This leads to an increase in the range where $S$-financing is optimal. The same is also true for an increase in $p_S$. 
to a high level of support, but ex post it cannot promise to deliver it. Achieving synergies can thus become a competitive hinderance.

4. Valuation and value

The analysis so far focuses on the question of who will get to finance the new venture. We now use the model to make predictions about the equity stakes offered in equilibrium. In the venture capital industry, offers are measured in terms of their implied valuation of the firm. The so-called post-money valuation calculates the value of the firm on the basis of the stake purchased by the investor. If an investor pays an amount $I$ to obtain a stake $a$, then the valuation $G$ is given by

$$ G = \frac{I}{a} $$

The so-called pre-money valuation is given by

$$ G = \frac{I}{\frac{1}{C_0}a + \frac{1}{C_0}p} $$

and behaves analogously. The larger $a$, the more the entrepreneur has to give up to raise an amount $I$, and thus the lower the valuation of the new venture.

When $E$ chooses an investor, let $\Gamma_S$ and $\Gamma_V$ be the highest valuation that she can obtain from $S$ or $V$. Furthermore, let $\Gamma_M = I/(z_S^M + z_V^M)$ be the highest valuation under mixed financing. Finally, define $\theta_I = (P_V^2 - P_S^2 + 2I(p_V^2 - 2p_S^2))/2p_S^2\sqrt{P_V^2 + 2Ip_V^2}$.

Fig. 5. An increase of $P_V$ from $P_V^l$ to $P_V^u$ increases the efficient effort level under $V$-financing, but not under $S$-financing. It also increases the optimal effort for $V$ but not for $S$. This leads to an increase in the range where $V$-financing is optimal. The same is true for an increase in $p_V$. 
**Proposition 3 (Valuation).**

(i) If $S$ and $V$ are equally able, $E$ can always extract a higher valuation from $S$, i.e., $\Gamma_S > \Gamma_V$ for all $\theta \neq 0$.

(ii) $\Gamma_V$ is independent of $\theta$. $\Gamma_S$ is decreasing in $\theta$ for $\theta < \theta_F$ and increasing for $\theta > \theta_F$.

(iii) $\Gamma_V$ is increasing in $P_V$ and $p_V$. $\Gamma_S$ is increasing in $P_S$ and $p_S$.

(iv) For $\theta < \theta_0$, mixed financing achieves a higher valuation than pure $V$-financing, i.e., $\Gamma_M > \Gamma_V$.

Fig. 6 illustrates $\Gamma_S$ and $\Gamma_V$, as a function of $\theta$. Consider first the case in which $S$ and $V$ are equally able. $\Gamma_V$ is constant, but $\Gamma_S$ lies above $\Gamma_V$ except for $\theta = 0$. Moreover, $\Gamma_S$ is decreasing for $\theta < 0$ and increasing in $\theta > 0$. Even if a corporation is an equally able investor, it will pay a higher valuation than an independent venture capitalist. Because $S$ cares about the externality $\theta$, $S$ is always keen to influence the new venture. Interestingly, this is true for complements and substitutes. For $\theta > 0$, $S$’s opportunity cost of not becoming the investor is that it will receive too little support. And for $\theta < 0$, $S$’s opportunity cost of not becoming the investor is that it will receive too much support. In either case, $E$ can extract a higher valuation from $S$ because of its strategic motivation. Fig. 6 also indicates the effect of increasing and decreasing $S$’s ability (i.e., changing $P_S$ and $p_S$). If $S$ becomes more able, $E$ can extract more value, and $S$ has to pay a higher valuation. But even if $S$ is
less able, he may still have to pay a higher valuation, either if \( \theta \) is sufficiently large and positive, or if \( \theta \) is sufficiently large but negative.

Proposition 3 also shows that over the range in which mixed financing is optimal, the valuation of the firm is higher under mixed financing than under pure \( V \)-financing. The intuition is that mixed financing improves \( S \)'s utility by preventing excessive cannibalization. In return for this benefit, \( E \) can again extract better terms from \( S \). This increases the overall valuation.

A key thing to recognize at this point is that valuation alone does not determine the investor choice. The following corollary emphasizes this point.

**Corollary to Propositions 1 and 3.** The entrepreneur does not always choose the investor from whom she could extract the highest valuation. A higher valuation may not imply a higher expected utility for the entrepreneur.

This corollary exposes the inadequacy of post-money valuations as a criterion for investment decisions. The valuations offered by different types of investors is a poor indicator of the attractiveness of the offers. This is because the valuation is not an indicator of the intangible benefit of having a value-added investor. To the contrary, it can be very misleading. As \( \theta \) extends further into the negative range, \( S \)'s valuation rises. However, since \( S \) provides increasingly less support, the utility of choosing \( S \) can actually decline. The correct decision criterion for making the investor choice is obviously \( E \)'s expected utility. All of our results also hold for \( \beta = 0 \). Maximizing \( U_E \) is thus equivalent to maximizing the expected returns of the insiders. The following proposition examines the structure of these expected returns.

**Proposition 4** (Non-monotonic returns).

(i) Suppose investors are equally able. \( E \)'s expected utility \( U_E(\theta) \) is decreasing for \( \theta < \theta_0 \), constant for \( \theta_0 < \theta < \theta_1 \), and increasing for \( \theta > \theta_1 \).

(ii) An increase in \( P_S \) or \( p_S \) increases \( U_E(\theta) \) for \( \theta > \theta_1 \). An increase in \( P_V \) or \( p_V \) increases \( U_E(\theta) \) for \( \theta < \theta_1 \).

This proposition shows that the expected utility has a nonmonotonic shape, as depicted in Fig. 7. The intuition is as follows. As a benchmark, consider the intermediate region \([\theta_0, \theta_1]\) in which \( V \)-financing is optimal. In this region \( U_E \) is constant, since \( \theta \) does not influence \( V \)'s support.

For \( \theta < \theta_0 \), \( E \) is better off with mixed financing, even though pure \( V \)-financing is still available. The lower \( \theta \), the better the terms that \( E \) can extract from \( S \) as a passive investor. Hence \( U_E(\theta) \) decreasing in \( \theta \). This result is particularly surprising. The lower \( \theta \), the more \( S \) dilutes \( V \)'s stake, and thus the lower the equilibrium probability of success \( q_V(\theta) = q^* \). Yet the lower \( \theta \), the higher \( E \)'s expected returns. This is because \( E \) can extract additional rents through the terms at which she sells a passive stake to \( S \).

A different rationale drives the result that \( U_E(\theta) \) is increasing for \( \theta > \theta_1 \). In this range, \( E \) can improve on pure \( V \)-financing by obtaining pure \( S \)-financing. The greater \( \theta \), the greater the value created by the new venture, and thus the greater the value captured by \( E \).
Proposition 4 has interesting implications for the choice of entrepreneurial projects. E is better off generating projects that create large externalities on S, be they positive or negative. The intriguing aspect is that it can be better to be a big threat, rather than a small threat. This is because only a big threat will force S to join the deal and allow E to extract additional rents.

The key insight here is that under a wide set of circumstances, S would pay a higher valuation than V. The post-money valuation, however, can be a misleading indicator of true firm values, since they do not account for the intangible benefits of support that different investors bring to the venture.

5. Extensions of the model

The previous two sections establish the main properties of the model. In this section we briefly consider a number of extensions. We focus on examining the robustness of the basic results, as well as pointing out some of the new insights. For simplicity we only consider the case in which S and V are equally able, i.e., $P_S = P_V = P$ and $p_S = p_V = p$.

5.1. The role of bargaining power

So far the equilibrium is based on a simple contracting game in which E can make a take-it-or-leave-it offer to S. If this offer is rejected, E has the option to obtain pure
V-financing from a competitive set of V’s. In this specification E has all the bargaining power vis-a-vis S, by virtue of making the take-it-or-leave-it offer. To examine how our results are affected by this specific assumption, we now consider a simple alternative model in which S has the bargaining power. Specifically, we assume that S can make a take-it-or-leave-it offer to E. S can offer either pure S-financing, or some type of syndicated financing. E can either accept the offer, or else take her outside option of pure V-financing.

**Proposition 5.** If S can make a take-it-or-leave-it offer, then

1. Propositions 1 and 2 continue to hold,
2. \( \Gamma_S \) is decreasing in \( \theta \), with \( \Gamma_S = \Gamma_V \) at \( \theta = 0 \); and
3. \( U_E \) does not depend on \( \theta \).

Part (i) says that the parameter regions in which S-financing, V-financing, or mixed financing are optimal do not depend on bargaining power. Intuitively, giving S more bargaining power changes \( x_S \), which affects the shape of the \( q_S(\theta) \) function in Fig. 1. However, \( q_S \) continues to be an increasing function of \( \theta \), it continues to intersect \( q_V \) at \( \theta_1 = 0 \), and the intersection of \( q_V \) with \( q^* \) continues to be at the same \( \theta_0 < 0 \). Therefore, the critical values between the three regions are not affected by relative bargaining power.

The main effect of bargaining power thus concerns valuation (i.e., Propositions 3 and 4). For \( \theta > 0 \), S can now obtain a lower valuation than V. S no longer has to pay for the fact that it is able to achieve synergies. Instead, S only has to ensure that the offer is attractive enough to prevent E from taking the outside option. The higher \( \theta \), the easier it is to get \( E \). Since S wants to provide more support, it can also offer a lower valuation to \( E \). This explains part (ii). For part (iii), we simply note that if S has all the bargaining power, then it can always hold down E to her reservation utility. This is given by E’s utility from pure V-financing, which is independent of \( \theta \).

This last result, however, depends on the extreme assumption that S has all the bargaining power. If we take a model in which E and S share bargaining power, we get back a nonmonotonic return function. In Appendix A, we consider the generalized Nash bargaining solution, which allows for a continuous parameterization of relative bargaining power. As long as E has some bargaining power \( U(\theta) \) has a nonmonotonic shape as depicted in Fig. 7. Moreover, the greater E’s bargaining power, the greater the absolute (i.e., both negative and positive) slope of \( U(\theta) \).

There are obviously many other variations of the contracting game that one could consider. In Appendix A, at the end of the proof of Proposition 5, we examine whether it matters that E approaches S before approaching V. We consider a simple model in which E can choose any sequence of offers, and in which it is never too late to make another offer. We show that such a model yields exactly the same outcome as our base model. Moreover, the exact sequence of offers does not matter in this model. Similarly, if the investors can make the offers, and it is never too late to make another offer, then the results from Proposition 5 apply. Appendix A also shows that if there is a last period to make an offer, then the order of offers may matter. The
results, however, remain very similar. The main difference is that for $\theta > 0$, $E$ prefers to wait until the last period before approaching $S$.

The main insight from this subsection is that the structure of the offer game affects bargaining power. This influences valuation, but not the overall contract structure.

5.2. Multiple strategic investors

So far the analysis assumed that there is only one strategic investor. The success of a new venture can affect several asset-owners. As a consequence, there can be several strategic investors that can all be characterized by their respective $\theta_i$’s. We will now extend the basic model and allow for two strategic investors.

We denote our two strategic investors by $S_H$ (high $S$) and $S_L$ (low $S$), and w.l.o.g. assume $\theta_H > \theta_L$. For simplicity, assume that $S_H$, $S_L$ and $V$ are all equally able. The efficient level of support is now given by $\sigma^* = p(1 + \beta + \theta_H + \theta_L)$. Define $\hat{\theta} = \alpha_{SH} - 1 - \beta < 0$, where $\alpha_{SH}$ satisfies $U^V_{SH}(\alpha_{SH}) = U^V_{SH}$ and $\hat{\theta} = \alpha_V - 1 - \beta < 0$, where $\alpha_V$ satisfies $U^V_{V}(\alpha_V) = 0$.

Proposition 6. Suppose $\theta_H > 0$, then

(i) for $\theta_L > \hat{\theta}$, we have pure $S_H$-financing with $\sigma_{SH} < \sigma^*$
(ii) for $\theta_L < \hat{\theta}$, we have mixed financing, where $S_H$ is the active investor, $S_L$ the passive investor, and $\sigma_{SH} = \sigma^*$.
Suppose $\theta_H < 0$, then
(iii) for $\theta_L + \theta_H > \hat{\theta}$, we have pure $V$-financing with $\sigma_V < \sigma^*$
(iv) for $\theta_L + \theta_H < \hat{\theta}$, we have mixed financing, where $V$ is the active investor, $S_L$ and $S_H$ are passive investors, and $\sigma_V = \sigma^*$.

This proposition shows how Proposition 1 can be extended to the case of multiple strategic investors. There always continues to be exactly one active investor. As long as the $S$ with the highest $\theta$ is a complement (i.e., $\theta_H > 0$), it is chosen to be the active investor. Otherwise $V$ becomes the active investor. Note that since $V$ is an investor with $\theta = 0$, $V$ effectively becomes the investor with the highest $\theta$. Proposition 6 also shows that the role of passive investors can be shared by several strategic investors. For $0 > \theta_H > \theta_L$, both $S_H$ and $S_L$ could be asked to make investments without control.

We thus find that the basic structure of the model generalizes to the case with multiple investors. The main new insights are that the investor with the highest value of $\theta_i$ becomes the active investor, and there can be several passive investors.

5.3. Distinct investor abilities

So far we assumed that support activities are duplicative, i.e., that there is a set of tasks that can be performed by one investor. Support from a second investor does not affect the venture’s probability of success, since it only duplicates what the first investor has already done. Consider now a variation of the model in which $S$ and $V$
have distinct abilities, and in which the support provided by one investor does not subtract from the value of support by the other investor.

We can examine this using a simple extension of the model, in which we allow both investors to contribute to the success of the new venture. Assume that the probability of success is now given by \( q = P + p_a V + p_a S \).

**Proposition 7.**

(i) If \(|\theta|\) is not too large, then both \( S \) and \( V \) become active investors. The optimal contract satisfies \( z_V = z_S + \theta \), and the optimal \( z_S \) (\( z_V \)) is decreasing (increasing) in \( \theta \).

(ii) If \( \theta \) is sufficiently large and positive, then \( V \) is an active investor. \( S \) is an active partner, but without a stake in the company.

(iii) If \( \theta \) is sufficiently large and negative, then \( V \) is an active investor. \( S \) is not an active investor, though it can still be a passive investor.

Consider first the case in which \(|\theta|\) is not too large. In this case \( E \) wants to use both \( S \) and \( V \) as active investors, to benefit from their respective support. The optimal contract balances the incentives for \( S \) and \( V \), so that \( c_S(\theta) = p(z_S + \theta) = p, \quad c_V(\theta) = c_V(\theta). \) For larger \( \theta \), \( S \) will be allocated a smaller and \( V \) a larger equity stake. This means the stronger \( S \)'s strategic motivation, the less it requires monetary incentives. If \( \theta \) becomes sufficiently large and negative, the optimal contract has \( z_S = 0 \) and \( I_S = 0 \), i.e., \( E \) wants to allocate maximal equity to \( V \). She nonetheless keeps \( S \) as an active partner without equity stake, since \( S \) is strategically motivated to provide support. The situation is very different for the case when \( \theta \) is large and negative. In this case, giving control to \( S \) means giving \( S \) the opportunity to reduce the probability of success. Indeed, if \( z_S \) small, then \( \sigma_S = p(z_S + \theta) \) is negative, and \( S \) becomes destructive to the new venture. Obviously, \( E \) could increase \( z_S \), but there is a cost in terms of reducing \( V \)'s incentives. Instead, it can be better for \( E \) to simply refuse access to \( S \). In this case the model actually reduces back to the standard model with \( V \) as the only active investor. From our previous analysis, we know that for \( \theta < \theta_0 \), \( S \) becomes a passive investor.

The model in which investors have distinct abilities is fairly similar to the basic model. The main new aspect is that there can be a syndicate in which both investors are active. It can even be that \( S \) is an active partner, say with a board seat, but without an equity stake.

### 5.4. Endogenous strategic effects

So far, \( \theta \) is exogenous. By becoming an investor, \( S \) can gain additional means of influencing the new venture. \( S \) may want to reduce cannibalization or increase the complementarity of the new venture to its existing asset. In such an environment, the choice of investor can influence the level of \( \theta \). In particular, it can be that high values of \( \theta \) can be partly caused by choosing \( S \) as an investor. We now show that such endogeneity does not affect the basic results.
When S is not the active investor, we denote the default level of \( y \) by \( y^V \). We now assume that if \( S \) becomes an active investor, it can make additional investments to increase \( y \): \( S \) privately bears the cost of these investments. They are given by \( c(y) \) with \( c(y^V) > 0 \) and \( c''(y^V) < 0 \) for \( \theta > \theta_V \) and \( \psi(\theta) = 0 \) and \( \psi'(\theta) = 0 \) for \( \theta = \theta_V \). We denote \( S \)'s optimal choice of \( y \) by \( y_S \); which (using the envelope theorem) is given by \( q_S(y_S) = c'(y_S) \). It immediately follows that \( S \) always wants to increase \( y \); i.e., \( y_S > y^V \).

Proposition 8. If \( S \) can influence \( \theta \) by becoming an active investor, then the range of parameters in which \( E \) chooses pure \( S \)-financing becomes larger (relative to when \( \theta \) is exogenously fixed).

The intuition for this result is that if \( S \) is not the active investor, it forgoes the opportunity to influence \( \theta \) to his advantage. This increases his willingness to pay for becoming an active investor. As a consequence, \( E \) is also more likely to choose \( S \), and the range of pure \( S \)-financing increases.

This result is also useful for empirical testing. In particular, while the model predicts a positive correlation between \( \theta \) and \( S \)-financing, it does not depend on the direction of causality. In equilibrium, the positive correlation can be due both because higher \( \theta \)-firms choose \( S \)-financing and because \( S \)-financing further increases \( \theta \). This greatly facilitates empirical testing of the theory since we can test for simple correlations without having to disentangle the direction of causality.

5.5. Entrepreneurial effort

While the investor’s support is clearly important for the success of a new venture, it can be argued that the entrepreneur’s effort should also have a large, and possibly larger, impact on the likelihood of success. We thus extend the model to allow for double moral hazard, in which both \( E \) and \( S \) or \( V \) affect the performance of the new venture. In this case, there is a tradeoff between incentives for \( E \) versus incentives for the active investor. The most interesting aspect of this model is that the assumption of no ex ante side payments can be relaxed.

Suppose that \( E \) can provide effort \( \sigma_E \) at cost \( \frac{1}{2} \sigma^2_E \). This affects the probability of success, now given by \( q_i = P + p_E \sigma_E + p_i, i = S, V \), where we allow for \( p_E \neq p \).

Proposition 9. Suppose ex ante transfer payments are feasible, then \( \theta_0 = -\frac{1}{2} - \beta \) and \( \theta_1 = 0 \).

(i) If \( \theta > \theta_1 \), then there is pure \( S \)-financing. Relative to the first-best, there is too little support, i.e., \( \sigma_S < \sigma^*_S \) and \( \sigma_E < \sigma^*_E \).

(ii) If \( \theta_0 < \theta < \theta_1 \), then there is pure \( V \)-financing. Relative to the first-best, there is too little support, i.e., \( \sigma_V < \sigma^*_V \) and \( \sigma_E < \sigma^*_E \).

(iii) If \( \theta < \theta_0 \), then there is mixed financing, in which \( V \) is the active and \( S \) is the passive investor. The level of support is first-best, i.e., \( \sigma_V = \sigma^*_V \) and \( \sigma_E = \sigma^*_E \).
The basic intuition is that with a double moral hazard, there is a team problem in which both $E$ and the active investor would invest more efficiently if they had a larger share of the total returns $(1 + \beta + \theta)$. In this setting it continues to be true that $S$ is the more efficient investor for $\theta > 0$, since there are more total incentives to go around between $E$ and $S$ than between $E$ and $V$ (i.e., $1 + \beta + \theta > 1 + \beta$). And for $\theta < 0$, there are more total incentives to go around between $E$ and $V$ than between $E$ and $S$ (i.e., $1 + \beta > 1 + \beta + \theta$). In this case $V$-financing is more efficient. If, however, there is overinvestment under pure $V$-financing, then $S$ can absorb some equity as a passive investor.

The main insight from Proposition 9 is that the basic insights are not affected by entrepreneurial effort, and that with this extension, it is also possible to relax the assumption of no ex ante transfer payments.

6. Conclusion

What is the difference between a strategic and a purely financial venture investor? The venture capital market is not only growing significantly in terms of size and economic importance, it is also witnessing the entry of a large diversity of new investor types, such as corporations, banks, and other established organizations. Identifying the fundamental differences between various types of venture investors is central to an understanding of entrepreneurial finance.

This paper provides a theoretical foundation for analyzing the competitive advantages and disadvantages of strategic investors. It explains when entrepreneurs prefer investors that operate in related segments, and when they prefer independent venture capitalists. The analysis generates a large number of testable implications. If the new venture is a complement to the asset of the strategic investor, then the strategic investor is more likely to finance the new venture. If it is a substitute, but poses only a small threat to the strategic investor, then venture capitalist financing is more likely. However, if the venture poses a bigger threat to the strategic investor, the model predicts syndicated finance where the venture capitalist holds board seats and control rights and the strategic investor is relegated to a more passive role. If there are several strategic investors that are all complements, then the strategic investor with the highest complementarities should finance the new venture. If several are substitutes, they may all become passive investors. If the strategic investor is better able to provide support to a venture, then he may finance it even if it is a substitute. But if he is less able, the venture capitalist can finance the venture even if it is a complement. And if the two investors bring different skills to the table, there can be syndicated finance with both types of investors holding boards seats and/or other control rights.

The paper also makes some predictions about the relative valuations that a strategic investor and a purely financial investor would offer for a given investment opportunity. If the new venture is a complement and the entrepreneur has a lot of (little) bargaining power, then the strategic investor would pay a higher (lower) valuation. If the new venture is a substitute, then the strategic investor always pays a
higher valuation, regardless of bargaining power. Moreover, syndicated financing commands a higher valuation than pure venture capital financing.

A central insight of the analysis is that the strategic investor’s quest for synergies can turn into a competitive disadvantage. The next logical step in this line of research would be to examine how strategic investors can create structures that partially overcome these problems. Future research might want to focus on issues of organizational design and the compensation structure of venture managers. These issues have recently reached a new level of prominence as a large number of companies are actively looking for a comprehensive strategy for investing in entrepreneurial companies. Some companies, like Cisco, favor an approach where the venture capital activity is closely integrated with the core business. Others like Texas Instruments have essentially outsourced all their venture capital investments with minimal organizational control. Further deepening our understanding of the strategic investment process is thus highly relevant from an academic as well as a practical perspective.

Appendix A. Proofs

Proof of Propositions 1 and 2. Consider first pure V-financing. E maximizes \( U^V_E \), subject to \( U^V_E = (P_V + p_V \sigma_E) x_V - \frac{1}{2} \sigma^2_V - I = 0 \) and the first order condition \( \sigma_V = p_V x_V \). Using \( x_V = 1 \), we recognize that the project is feasible as long as \( I \leq P_V + \frac{1}{2} p^2_V \). We can solve for \( x_V \) in \( U^V_E = 0 \) to obtain \( x_V = (\sqrt{P^2_V + 2 p^2_V I} - P_V) / p^2_V \). It will be useful for later to note that \( q_V = P_V + p_V \sigma_V = \sqrt{P^2_V + 2 p^2_V I} \), \( U^V_S = y_S + q_V \theta \), and \( U^V_E = q_V (1 + \beta - x_V) \). Also note that \( d x_V / d I > 0 \), \( d x_V / d P_V < 0 \) and \( d x_V / d P_V < 0 \).

Consider next pure S-financing. S’s outside option is that V-financing goes ahead. E thus chooses \( x_S \) to maximize \( U^S_E \) subject to \( U^S_S = U^V_S \), and subject to the first order condition \( \sigma_S = p_S (x_S + \theta) \). Solving for \( x_S \) in \( U^S_S = U^V_S \) we get after transformation \( x_S + \theta = (\sqrt{P^2_S + 2 p^2_S (I + \theta q_V)} - P_S) / p^2_S \). It follows that \( \sigma_S = p_S (x_S + \theta) \) is an increasing function of \( \theta \). Obviously, \( E = p_V x_V \) is independent of \( \theta \). Moreover, for \( P_S = P_V \) and \( p_S = p_V \), we have \( \sigma_S = \sigma_V \) at \( \theta = 0 \).

Consider also mixed financing in which \( V \) is the active investor. Let \( I = I^M_S + I^M_V \). E chooses \( x^M_V \), \( x^M_S \), \( I^M_S \), and \( I^M_V \) to maximize \( U^V_E \) subject to \( U^V_M = 0 \) and \( U^S_M = U^V_S \) and subject to the first-order condition \( \sigma_V = p_V x^M_V \). Since the contract that maximizes \( U_E \) also maximizes \( W \), it is immediate that mixed financing is better than V-financing whenever \( \sigma_V > \sigma^*_V \iff x_V > 1 + \beta + \theta \). We have \( \theta_0 = x_V - 1 - \beta \). Using this and the comparative statics of \( x_V \), we immediately recognize \( d \theta_0 / d \theta < 0 \), \( d \theta_0 / d I > 0 \), \( d \theta_0 / d P_S = d \theta_0 / d P_V = 0 \), \( d \theta_0 / d P_V < 0 \), and \( d \theta_0 / d P_V < 0 \).

To see that E prefers S-financing over V-financing for \( \theta > \theta_1 = 0 \), simply note that for \( \sigma < \sigma^V \), W is an increasing function of \( \sigma \). For \( \theta > \theta_1 \), we have \( \sigma^* > \sigma_S > \sigma_V \) and for \( \theta_0 < \theta < \theta_1 \), we have \( \sigma^* > \sigma_S > \sigma_S \). Increases in \( \beta \) or \( I \) do not affect the fact that \( \sigma_S = \sigma_V \) at \( \theta = 0 \). However, an increase in \( P_S \) or \( p_S \) implies \( \sigma_S > \sigma_V \) for \( \theta = 0 \). It follows that \( \theta_1 < 0 \). Note also that \( \theta_1 = \theta \), where \( \sigma_S(\theta) = \sigma_V \), since switching to V now also implies an efficiency loss, i.e., \( W_S(\sigma_S(\theta)) > W_V(\sigma_V) \). Similarly, an increase in \( p_V \).
or \( P_V \) implies \( \sigma_V > \sigma_S \) for \( \theta = 0 \), so that \( \theta_1 > 0 \). Moreover, \( \theta_1 > \hat{\theta} \), since switching to \( V \) now also implies an efficiency gain.

It is now also easy to see that it is never optimal to have \( S \) as the active investor and \( V \) as a passive investor. This is simply because the level of support with this inverted type of mixed financing always lies below \( \sigma_S \), which itself lies below the efficient level \( \sigma^*_S \).

The proof uses the property that the contract that maximizes \( U_E \), subject to \( \theta_1 = 0 \) and \( U_S = \dot{U}_S = U'_S \), also maximizes \( W = U_E + U_V + U_S \), subject to the same constraints. This requires that all participation constraints are binding. We need to check that \( E \) would never give any investor any additional equity, to improve incentives. Consider \( U'_E = (P_V + p_V^2x_V)(1 + \beta - \alpha_V) \), so that \( \frac{dU'_E}{dx_V} = p_V^2(1 + \beta) - p_V - 2p_V^2x_V \). This is always negative for \( P_V > p_V^2(1 + \beta) \). Similarly, consider \( \dot{U}_E^S = (P_S + p_S^2(\alpha_S + 0))(1 + \beta - \alpha_S) \), so that \( \frac{dU'_E}{dx_S} = \dot{P}_S^2(1 + \beta - \theta) - P_S - 2p_S^2z_S \), which is always negative for \( P_S > \dot{P}_S^2(1 + \beta - \theta) \).

Proof of Proposition 3. We begin with the comparative statics. It is immediate to see that \( \frac{dx_i}{dp_i} < 0 \) and \( \frac{dx_i}{dp_i} < 0 \), \( i = S, V \), so that \( \Gamma_i = I/x_i \) increasing in \( P_i \) and \( p_i \). To see how \( \Gamma_S \) depends on \( \theta \), rewrite \( \alpha_S = -\theta - P/p_S^2 + \frac{x_S}{p_S^2} \), where \( \xi = \sqrt{\dot{P}_S^2 + 2p_S^2(I + \theta q_V)} \). Consider \( \frac{d\alpha_S}{d\theta} = -1 + (1/p_S^2)(1/2\xi)2p_S^2q_V = q_V/\xi - 1 \). But \( q_V > \xi \Leftrightarrow P_V^2 + 2p_V^2I > P_S^2 + 2p_S^2(I + \theta \sqrt{\dot{P}_V^2 + 2lp_V^2}) \) so that

\[
\frac{d\alpha_S}{d\theta} > 0 \Leftrightarrow \theta < \frac{P_V^2 - P_S^2 + 2I(p_V^2 - p_S^2)}{2p_V^2 \sqrt{\dot{P}_V^2 + 2lp_V^2}} = \theta_f. \tag{9}
\]

Next, compare \( \Gamma_V \) and \( \Gamma_S \) at \( P_S = P_V = P \) and \( p_S = p_V = p \). We claim that for \( \theta \neq 0 \) we have \( \Gamma_S > \Gamma_V \Leftrightarrow \alpha_S < \alpha_V \Leftrightarrow -\theta + (\sqrt{P^2 + 2p^2(I + \theta q_V)} - P)/p^2 < (q_V - P)/p^2 \). After transformations we obtain \( P^2 + 2p^2I < q_V^2 + (p^2\theta)^2 \), and using \( q_V^2 = P^2 + 2p^2I \) we obtain \( P^2 + 2p^2I < P^2 + 2p^2I + (p^2\theta)^2 \Leftrightarrow 0 < (p^2\theta)^2 \). This implies \( \Gamma_S > \Gamma_V \) for \( \theta \neq 0 \).

To see that \( \Gamma_V < \Gamma_M = I/(\alpha^M_S + \alpha^M_V) \), consider \( \alpha_V \) from pure \( V \)-financing, and suppose that \( \alpha^M_S + \alpha^M_V = \alpha_V \). We will show that \( S \) is better off with this particular contract, i.e., \( U^M_S > U'_S \). To see this, use \( \alpha^M_V = (1 + \beta + \theta) \) to obtain

\[
U^M_S - U'_S = (P_V + p_V^2(1 + \beta + \theta))(\alpha_V - (1 + \beta + \theta)) - (I - I_V)
+ [(P_V + p_V^2(1 + \beta + \theta))(\alpha_V - (1 + \beta + \theta)) - (P_V + p_V^2x_V)\alpha_V] \theta. \tag{10}
\]

From \( U^M_V = 0 \) we obtain \( I_V = p_Vx_V + (p_V^2/2)(\alpha_V^M)^2 = P_V(1 + \beta + \theta) + (p_V^2/2)(1 + \beta + \theta)^2 \) so that \( (I - I_V) = P_Vx_V + (p_V^2/2)(\alpha_V^M)^2 - P_V(1 + \beta + \theta) - (p_V^2/2)(1 + \beta + \theta)^2 \). After some transformations, the first two terms of \( U^M_S - U'_S \) are thus given by

\[-(p_V^2/2)((1 + \beta + \theta) - \alpha_V)^2 < 0 \]. This says that when \( S \) takes a passive stake, it becomes less valuable than under pure \( V \)-financing. This is because \( V \) now provides less support. But \( S \) also gets a utility from the lower support, as measured by the third term, which simplifies to \( p_V^2(((1 + \beta + \theta) - \alpha_V)\theta > 0 \). Putting these terms together, we obtain

\[
U^M_S - U'_S = \frac{1}{2}p_V^2(\alpha_V - (1 + \beta + \theta))(1 + \beta + \theta - \alpha_V - 2\theta) \]. 
Take \( \theta_0 \), where \( \sigma^M = \sigma_V \). At \( \theta_0 \), we have \( (1 + \beta + \theta_0) = \alpha_V \), so that \( U^M_S - U'_S = 0 \). For all
If $\theta < \theta_0$ we have $1 + \beta + \theta = 1 + \beta + \theta_0 - (\theta_0 - \theta) = \alpha_V' - (\theta^* - \theta)$ so that $U_S^M - U_V^S = \frac{1}{2} p_V'(\theta^* - \theta) - \theta - \theta^* - \theta_0$, which thus proves $U_S^M > U_V^S$. This says that the particular offer we considered is too generous to $S$. In equilibrium, $E$ thus offer $S$ less equity, i.e., $\alpha_S^M < \alpha_V^M \iff \alpha_V^M > \alpha_S^M$ so that $\Gamma V > \Gamma M$.

Proof of Proposition 4. Consider $U_E^S = (P_S + p_S^2(\alpha_S + \theta))(1 + \beta - \alpha_S)$. Using $\alpha_S = -\theta - P/p_S^2 + \xi/p_S^2$ where $\xi = \sqrt{p_S^2 + 2p_S^2(I + \theta q_V)}$, we have $d\alpha_S/d\theta = \sqrt{p_S^2 + 2p_S^2I}/\sqrt{p_S^2 + 2p_S^2(I + \theta q_V)} - 1 = q_V/q_S - 1$ and thus

$$
\frac{dU_E^S}{d\theta} = (P + p_S^2(\alpha_S + \theta))(1 - \frac{q_V}{q_S}) + p_S^2(1 + \beta - \alpha_S)\frac{q_V}{q_S}.
$$

With equal abilities, we have $q_S > q_V$ for $\theta > \theta_1 = 0$, and thus $dU_E^S/d\theta > 0$. For $\theta_0 < \theta < \theta_1$, we have $U_E^S = (P_V + p_V^2(\alpha_V + \theta))(1 + \beta - \alpha_V)$, which is independent of $\theta$. And to see that $U_E(\theta)$ is decreasing for $\theta_0 < \theta$, note from $W_M = U_M^S + U_V^S + U_M^S$ and $U_M^M = 0$, that $U_E(\theta) = W_M - U_M^S$. Using $U_M^S = U_M^S = u_S + \theta \sqrt{p_S^2 + 2P V^2}$ and $W_M = (P_V + p_V^2(1 + \beta + \theta))(1 + \beta + \theta) - p_V^2(1 + \beta + \theta)^2/2 + u_S - I$, we obtain $U_E(\theta) = P_V(1 + \beta + \theta) + p_V^2(1 + \beta + \theta)^2/2 + u_S - I - \theta \sqrt{p_V^2 + 2P V^2}$ and thus $dU_E/d\theta = P_V + p_V^2(1 + \beta + \theta) - \sqrt{p_V^2 + 2P V^2}$.

Similarly, we get $dU_E/\theta < 0$ for $\sigma_V > \sigma^*_V$. When mixed financing is optimal, $U_E(\theta)$ is therefore decreasing in $\theta$.

To see that $U_E(\theta)$ increasing in $p_V$ and $P_V$ for $\theta < \theta_1$, consider first $\theta < \theta_0$. In this case, $dU_E/dP_V = (1 + \beta + \theta) - \theta P_V/\sqrt{p_V^2 + 2P V^2} > 0$ and $dU_E/dp_V = P_V(1 + \beta + \theta) - \theta P_V/\sqrt{p_V^2 + 2P V^2} > 0$, where we use $\theta_0 < - \beta < 0$. For $\theta_0 < \theta < \theta_1$, $U_E(\theta) = U_E(\theta) = (P_V + p_V^2(\alpha_V + \theta))(1 + \beta - \alpha_V)$, so that $dU_E(\theta)/d\alpha_V = (dU_E(\theta)/d\alpha_V)(d\alpha_V/dP_V) + 2p_V\alpha_V(1 + \beta - \alpha_V) > 0$ since $dU_E(\theta)/d\alpha_V < 0$ and $d\alpha_V/dP_V < 0$. Similarly, $dU_E(\theta)/dP_V = (dU_E(\theta)/d\alpha_V)(d\alpha_V/dP_V) + \alpha_V(1 + \beta - \alpha_V) > 0$ since $d\alpha_V/dP_V < 0$.

Proof of Proposition 5. Suppose now that $S$ can make a take-it-or-leave-it offer to $V$. If $E$ rejects this offer, she simply obtains a competitive bid for pure $V$-financing. As before, we denote this bid by $\alpha_V^M$. It satisfies $U_V^M(\alpha_V^M) = 0$, and we denote $E$’s resulting utility by $U_E^M(\alpha_V^M) \equiv U_E^M$. $S$’s bid, denoted by $\alpha_S^M$, satisfies $U_E^M(\alpha_S^M) \equiv U_E^M$. Note that we can think of the case in which $S$ makes no bid a special case in which $S$ proposes $\alpha_S^M = 0, \alpha_V^M = \alpha_V^M$. $S$’s optimal offer thus maximizes $U_S = \text{Max}[U_S^M, U_S^M]$, subject to $U_E^M = U_E^M$ and $U_V^M = 0, i = S, V$. Maximizing $U_S^M$ is then equivalent to maximizing $U_S^M + U_V^M + U_E^S = U_S^M + U_V^S + U_E^M = W_i$, so that $S$’s optimal offer always maximizes $W = \text{Max}[W_S, W_V]$. This maximization problem is very similar to before.

For part (i), we need to show that $q_S$ is again an increasing function of $\theta$. To see this, differentiate $U_E^M(\alpha_S^M, \theta) = U_E^M$, to get $(\partial U_E^M/\partial \alpha_S^M) d\alpha_S^M + (\partial U_E^M/\partial \theta) d\theta = 0$. Using
(\partial U_S^E / \partial x_S^E) = p_S^2(1 + \beta - x_S^E) - [P_S + p_S^2(x_S^E + \theta)] < 0 \quad (\text{since } P_S > (1 + \beta - \theta)p_S^2) \text{ and } \\
(\partial U_E^E / \partial \theta) = p_S^2(1 + \beta - x_S^E) > 0 \quad \text{we get } \frac{d\xi^E_S}{d\theta} = -p_S^2(1 + \beta - x_S^E)/(P_S + p_S^2(x_S^E + \theta) - p_S^2(1 + \beta - x_S^E)) > 0. \quad \xi_S^E \text{ is thus an increasing function of } \theta. \text{ With } q_S = P_S + p_S(x_S^E + \theta), \text{ it follows that } q_S \text{ is also an increasing function of } \theta. \text{ At } \theta = 0 \text{ we naturally have } q_S = q_V. \text{ Since the optimal offer again maximizes the joint surplus } W, \text{ the remainder of the analysis is analogous to the proof of Propositions 1 and 2.}

For part (ii), we simply use \(d\xi^E_S/d\theta > 0\) to note that \(d\Gamma^E_S/d\theta = -(1/(\xi^E_S)^2)(d\xi^E_S/d\theta) < 0\). Moreover, at \(\theta = 0\), we have \(\xi^E_S = \xi^V_S\), so that \(\Gamma^E_S = \Gamma^V\).

For part (iii) we simply note that the condition \(U^1_E(\xi^E_S) = U^E_E\) is binding for all values of \(\theta\), and that \(U^E_E\) is independent of \(\theta\).

To see that \(U^E_E(\theta)\) is independent of \(\theta\) only in the extreme case in which \(E\) has no bargaining power, we examine the generalized Nash bargaining solution. In this case, \(E\) and \(S\) maximize \(W^E\), subject to their outside options, given by \(U^E_E(\xi^V_S) = U^E\) and \(U^E_S(\xi^V_S) = U^S(\theta)\). Let \(W(\theta) = W(\theta) - U^E - U^S(\theta)\) be the net surplus, and \(\tau_E\) be \(E\)'s bargaining strength, then \(U^E_E(\theta) = U^E + \tau_E W(\theta) = (1 - \tau_E)U^E + \tau_E [W(\theta) - U^S(\theta)]\).Clearly, \(U^E_E\) is independent of \(\theta\) only for \(\tau_E = 0\). Moreover, for \(\tau_E = 1\), we have \(U^E_E(\theta) = W(\theta) - U^S(\theta)\), so that \(W(\theta) - U^S(\theta)\) has the same shape as in Proposition 4 (Fig. 7). Its slope is given by \(dU^E_E(1)/d\theta = dW(\theta)/d\theta - dU^S(\theta)/d\theta\). It follows that \(U^E_E(\theta) = (1 - \tau_E)U^E + \tau_E [W(\theta) - U^S(\theta)]\) also has the same shape, except that the slopes of \(U^E_E(\theta)\) are reduced by a factor \(\tau_E\), i.e., \(dU^E_E(\theta)/d\theta = \tau_E dW(\theta)/d\theta - dU^S(\theta)/d\theta = \tau_E dU^E(1)/d\theta\).

We will now discuss the importance of offer sequence. Our model so far assumes that \(E\) approaches \(S\) before approaching the market of \(V\)'s. Consider now a more general model in which \(E\) can make offers in any sequence. We also assume that there is no last date, i.e., it is never too late to make another offer. Suppose that \(E\) always makes the offers that we derived in the base model. Every \(V\) will always accept an offer \(\xi^V_S\), since \(E\) will never want to accept a better offer. Similarly, \(S\) will always accept the equilibrium offers \(\xi^E_S\), \(i = S, V\), that we derived for the base model. \(S\) knows that \(E\) can always get pure \(V\)-financing, and knows that \(E\) will never want to make a better offer either. In this model it does not matter whether \(E\) approaches \(S\) first. Indeed, for pure \(V\)-financing \(E\) may as well approach \(V\) directly, since she will not make an acceptable offer to \(S\) anyway.

As long as \(E\) is making the offers, it is easiest to assume that \(E\) makes one offer at a time. This way there is no problem that several investors might want to accept different contracts at the same time. If investors are making the offers, and offers can be made at any time, then there can also be simultaneous offers. In any period, \(E\) can either choose to accept one offer or none. Consider the model in which the investors can make offers at any time and in which it is never too late to make further offers. All the \(V\)'s are willing to make competitive bids \(\xi^V_S\) at any time. And \(S\) is willing to make the same bid as in Proposition 5 at any date too. The results from Proposition 5 thus continue to hold, and the exact order of bids does not matter.

Consider next final period effects. If investors make the offers, then it is easy to see the model is as before. This is because all the investors are always willing to make the above bids even on the final period. We therefore focus on the case in which \(E\) makes the offers. \(V\) behaves just as before in the last period. But \(S\) has a different outside
option, since refusing the offer now terminates the venture. This changes \( S \)'s reservation utility from \( U_S = y_S + q_V \theta \) to \( U'_S = y_S \). For \( \theta > 0 \), \( S \)'s reservation utility is now lower, so that \( E \) can extract a higher valuation than before. Disregarding any costs of delay, \( E \) would thus actually wait for the last period. This yields a higher equilibrium valuation, given by \( \Gamma = I/\alpha_S \), where \( \alpha_S \) satisfies \( P_S(\alpha_S + \theta) + \frac{1}{2} p_S(\alpha_S + \theta)^2 = I_S \).

For \( \theta < 0 \), \( S \)'s reservation utility is now higher. In the last period, \( E \) will always choose pure \( V \)-financing, rather than syndicated finance. To see this, consider \( \alpha_V \) from pure \( V \)-financing, and consider mixed financing at the same valuation, i.e., \( \alpha^M_S + \alpha^M_V = \alpha_V \). In the proof of Proposition 3, we showed that \( S \) would be better off with mixed financing at the same valuation. With an altered outside option, however, we will now show that \( S \) is worse off. We have \( U^M_S = (P_V + \frac{1}{2} \alpha^2_V (\alpha_V - \alpha^M_V))(\alpha^M_S + \theta) - (I - I_V) \). From \( U^M_S = 0 \) we obtain \( I_V = P_V \alpha^M_V + p^2 \alpha^M_V / 2 \) so that \( (I - I_V) = P_V \alpha_V + (\frac{1}{2} p^2 / 2) \alpha^2_V - P_V (\alpha_V - \alpha^M_V) - (\frac{1}{2} p^2 / 2) (\alpha_V - \alpha^M_V)^2 \). After transformations we obtain \( U^M_S = y_S + (P_V + \frac{1}{2} \alpha^2_V (\alpha_V - \alpha^M_V)) \theta - (\frac{1}{2} p^2 / 2) (\alpha^M_V)^2 < y_S \). This says that \( S \) is worse off. To have mixed financing, \( E \) would have to give up proportionally more equity, i.e., \( \alpha^M_S > \alpha_V - \alpha^M_V \). But this means that mixed financing makes \( E \) worse off on two counts: lower support from \( V \) and greater ownership dilution. In the last period, \( E \) would thus prefer pure \( V \)-financing. Prior to the last period, however, \( E \) can still approach \( S \). \( S \)'s outside option is then given by what would happen in the last period, namely pure \( V \)-financing. \( E \) can thus propose the same mixed finance contract as before, and we are back to the base model. The equilibrium for \( \theta < 0 \) is thus the same as in the base model, even if there is a last period in the game.

These results show that if either the entrepreneur, or the investors, make all offers, the actual sequencing of offers does not matter very much, except for final period effects. Obviously, one could further complicate the game structure, such as by having \( E \), \( V \) and \( S \) all make offers at different times. Recent work by Hart and Mas-Colell (1996), Hellmann (2000), and Marx and Shaffer (2000) suggests that the sequencing of offers might affect the distribution of bargaining power in such multi-party games. More complex offer structures might thus influence the valuation, although our previous analysis also suggests that it need not affect the overall contracting structure.

Proof of Proposition 6. Suppose that \( E \) can make a take-it-or-leave-it to any subset of \( \{S_H, S_L, V\} \), and if the offer is rejected, \( E \) seeks funding from some other \( V \)'s. The proof is then a direct extension of Proposition 1. Suppose here that \( \theta_H > 0 \). The proof with \( \theta_H < 0 \) is analogous, except that we replace \( S_H \) with \( V \), which implicitly has \( \theta = 0 \). Using Proposition 1, \( S_H \) is preferred to \( V \). If \( \theta_L > 0 \), we can see from Proposition 4 that \( S_H \) is preferred to \( S_L \) because \( U_E^S \) is increasing in \( \theta \). This continues to be true for \( 0 > \theta_L > \theta_H \), where we still have \( \sigma^S_H < \sigma^* \). For \( \theta_L < \theta_H \), however, we get \( \sigma^S_H > \sigma^* = p(1 + \beta + \theta_H + \theta_L) \). We only need to verify that \( \sigma^* = p(1 + \beta + \theta_H + \theta_L) \) is the relevant threshold for mixed financing when \( S_H \) is active and \( S_H \) is passive. Consider \( \alpha^M_S \) so that \( S_H \) is indifferent between \( U^M_{S_H} \) and \( U^V_{S_H} \), i.e., \( P(\alpha^M_S + \theta_H) + p^2 (\alpha^M_S + \theta_H)^2 / 2 - I_{S_H} = \theta_H q_V \), where \( q_V = \sqrt{p^2 + 2I p^2} \). Solving the quadratic equation we have \( \alpha^M_S + \theta_H = -p / p^2 + q_{S_H} / p^2 \), where \( q_{S_H} = \)}
Eqs. (6) and (7) simultaneously. Let emphasize that both investors are active. The first order conditions are now given by

$$p - p = a \quad d$$

Consider an increase in inequality constraints are binding, which only occurs for

$$ob \quad these in$$

$$SL \quad UM$$

Solving the quadratic equation we have

$$a = \sqrt{p^2 + 2p^2(I_{SH} + \theta qV)}.$$ Consider next $z_M^M$ and $I_{SL}$ so that $S_L$ is indifferent between $U_M$ and $U_V$. We have $q_{SH}(z_M^M + \theta L) - I_{SL} = qV \theta L$ so that $z_M^M + \theta L = (qV \theta L + I_{SL})/q_{SH}$. We use this in $U_E^M = (P + p^2(z_M^M + \theta H))(1 + \beta - z_M^M - \beta_{SH}) = qSH(1 + \beta + \theta H + \theta L) + q_{SH}P/p^2 - q_{SH}^2/pL - qV \theta L - I_{SL}$. We have

$$dU_E/ds_{SL} = -1 + (dU_E^M/dqV)(dqV/ds_{SL}) = 1 - (P^2/q_{SH})(1 + \beta + \theta H + \theta L) - P/q_{SH},$$ so that $dU_E^M/ds_{SL} > 0$ if $q_{SH} > P + p^2(1 + \beta + \theta H + \theta L)$ or $\sigma_{SH} > p(1 + \beta + \theta H + \theta L) = \sigma^*$. Thus $\sigma^*$ is the relevant threshold.

To complete the proof, we need to examine whether $S_L$ and $S_H$ are both used as a passive investors. Consider $z_M^M$ so that $U_M^M = U_V^M = 0$, i.e., $Pz_M^M + p^2z_M^M/2 - I_V = 0$. Solving the quadratic equation we have $z_M^M = -P/p^2 + qV/p^2$, where $qV = \sqrt{p^2 + 2p^2I_V}$. Consider next $z_M^M$ and $I_{SL}$, so that $S_L$ is indifferent between $U_M$ and $U_V$. We have $(P + p^2z_M^M)(z_M^M + \theta L) - I_{SL} = qV \theta L$ so that after transformation $(z_M^M + \theta L) = (qV \theta L + I_{SL})/q_M^M$. Similarly, $(z_M^M + \theta H) = (qV \theta H + I_{SH})/q_M^M$. We use these in $U_E^M = (P + p^2z_M^M)(1 + \beta - z_M^M - z_M^M - \beta_{SH}) = qV^2P/p^2 - qV^2/p^2 - qV \theta H - I_{SH}$. We have

$$dU_E^M/ds_{SL} = -1 + (dU_E^M/dqV)(dqV/ds_{SL}) = 1 - (P^2/qV^2)(1 + \beta + \theta H + \theta L) - P/qV^2,$$ so that $dU_E^M/ds_{SL} > 0$ if $qV > P + p^2(1 + \beta + \theta H + \theta L)$ or $\sigma_M^M > (1 + \beta + \theta H + \theta L)$. And similarly, $dU_E^M/ds_{SH} > 0$ whenever $\sigma_M^M > p(1 + \beta + \theta H + \theta L) = \sigma^*$. It follows that increasing $I_{SL}$ and $I_{SH}$ is beneficial whenever $\sigma_M^M > \sigma^*$, so that both $S_H$ and $S_L$ can become passive investors.

**Proof of Proposition 7.** $E$ will choose $z_V, z_S, I_V, I_S$ to maximize $W = q(1 + \beta + \theta - \sigma_Y^2)/2 - \sigma_z^2/2 - I$, subject to the participation constraints $U_M^M = U_V^M$ and $U_M^M = U_V^M = 0$. We use the subscript $J$ for joint finance (instead of $M$ for mixed finance) to emphasize that both investors are active. The first order conditions are now given by Eqs. (6) and (7) simultaneously. Let $q = (P + p^2z_V + p^2z_S + \theta)$, then $E$ maximizes

$$W = q(1 + \beta + \theta - p^2z_V^2/2 - p^2(z_S + \theta)^2/2 - I)$$

and $q(z_S + \theta)^2 - p^2(z_S + \theta)^2/2 - (I - I_V) - 3\sqrt{P^2 + 2Ip^2} = 0$, as well as the inequality constraints $I_V \geq 0$ and $I_V \leq I$. We associate $\lambda_V$ to first and $\lambda_S$ to second constraint. Increasing the Lagrangian with respect to $I_V$, we have $\lambda_V = \lambda_S = \lambda$, unless the inequality constraints are binding, which only occurs for $|\theta|$ sufficiently large. Consider an increase in $z_V$, then $dW/dz_V = p^2(1 + \beta + \theta - p^2z_V + \lambda[p + p^2z_V + p^2(z_S + \theta)]) = 0$ and $dW/dz_S = p^2(1 + \beta + \theta - p^2(z_S + \theta) + \lambda[p + p^2z_V + p^2(z_S + \theta)]) = 0$. It immediately follows that $z_V = z_S = \theta$. Moreover, the level of $z_S$ and $z_V$ is determined by the participation constraints $U_S^M = U_V^M$ and $U_S^M = U_V^M = 0$. We can use $z_V = z_S + \theta$, $q = P + 2p^2(z_S + \theta)$ and $qz_S - Pz_S = 0$ in $U_S^M = U_S^M$ to obtain after transformation $2P(z_S + \theta) + 3p^2(z_S + \theta)^2 - I - qz_S = 0$, where $q = \sqrt{p^2 + 2Ip^2}$. Solving this quadratic equation, we obtain $z_S + \theta = (1/3p^2)(-P + \sqrt{p^2 + (I + qz_S)3p^2})$. Replacing $z_V = z_S + \theta$, we immediately recognize that $dz_V/d\theta > 0$. Moreover, $dz_S/d\theta = -(1/3p^2)\sqrt{p^2 + 2Ip^2}3p^2/2\sqrt{p^2 + (I + qz_S)3p^2} < 0$ since $\sqrt{p^2 + 2Ip^2} < 2\sqrt{p^2 + (I + qz_S)3p^2}$ since $P^2 + 2Ip^2 < 4P^2 + 12Ip^2 + 120q^2p^2$.

The above contract holds for $|\theta|$ not too large. As $\theta$ becomes large and positive, $z_V$ is increasing and eventually $I_V \leq I$ becomes binding. In this case $z_S = 0$ and $S$ is
active, but without equity stake. As $\theta$ becomes large and negative, $x_V$ is decreasing and eventually $I_V \geq 0$ becomes binding, so that $x_V = 0$. But this supposes that both $S$ and $V$ are active investors. Consider the alternative of excluding $S$, so that $\sigma_S = 0$.

To see how exclusion of $S$ can be beneficial, compare $W_J$ for joint financing, with $W_V$ for pure $V$-financing. Keeping the participation constraints always binding, we can decompose $W_J - W_V$ into two components. First there is the net benefit of giving $S$ access (without any financial investments or stakes), which is given by $W_J(\alpha_S = 0) - W_V$. And second, there is the net benefit of giving $S$ optimal incentives through a financial investment and stake, which is given by $W_J(\alpha_S) - W(\alpha_S = 0)$, where $\alpha_S$ is the optimal choice. If $\theta > 0$, this $W(\alpha_S = 0) > W(V)$ and $W(\alpha_S = \alpha_V - \theta) > W(\alpha_S = 0)$, so that giving $S$ access is always beneficial. But for $\theta < 0$, it is possible that $W(\alpha_S = 0) < W(V)$. This is because giving $S$ access immediately triggers negative support of $\sigma_S = p_S \theta < 0$, i.e., giving access to $S$ creates potential for value destruction. Obviously, one can reduce or eliminate the negative support by giving $S$ some financial stake. While this incentive effect creates a positive net benefit, it may not be enough to offset the access effect which creates a negative net benefit. While the comparison of these effects is too difficult to trace analytically, we only need to provide a numerical example, to show that it is possible that the access effect dominates. In particular, we calculated the equilibrium contracts under the following parametric assumptions: $\beta = 0.2$, $p = 0.1$, $P = 0.5$, $u_S = 0$, and $I = 0.3$. All calculations were performed in Mathematica, and are available upon request. We obtained the following results:

As a well-behaved benchmark case, consider $\theta = 0.1$. If both $V$ and $S$ active we obtain $U^j_S = 0.0560$, $U^j_V = 0$, $U^j_E = 0.3072$, $x_S = 0.3470$, $x_V = 0.2470$ and $I_V = 0.1753$, whereas if only $V$ is active we obtain $U^j_S = 0.0560$, $U^j_V = 0$, $U^j_E = 0.3054$, $x_V = 0.5964$, which is less favorable for $E$.

To show that excluding $S$ can be optimal, consider $\theta = -0.9$. With both $V$ and $S$ active we obtain $U^j_S = -0.4554$, $U^j_V = 0$, $U^j_E = 0.3039$, $x_S = 0.5883$, $x_V = 0$ and $I_V = 0$. If only $V$ is active again we obtain $U^j_S = 0.0560$, $U^j_V = 0$, $U^j_E = 0.3054$, $x_V = 0.5964$. We immediately see that $U^j_V = 0.3039 < U^j_E = 0.3054$, so that having only $V$ active is better than having both $S$ and $V$ active.

To see that $S$ can still be useful as a passive investor, simply note that without access for $S$, the model reverts back to the standard model, and Proposition 1 applies. Note also that restricting access to $V$ is never necessary, since $V$ would never engage in negative support.

**Proof of Proposition 8.** If $\theta$ exogenous, the critical level where pure $S$-financing becomes better than pure $V$-financing, denoted by $\theta_1$, is given by $U^S(\sigma_S(\theta)) = U^S(\sigma_V)$. For any exogenous $\theta_V$, denote $S$’s equilibrium stake by $x_S(\theta_V)$. Allow $S$ now to make some investments in increasing $\theta$ from $\theta_V$ to $\theta_S$. At $x_S(\theta_V)$, this would increase $U^S_S$ from $U^S_S(x_S(\theta_V), \theta_V, \psi(\theta_V) = 0) = U^S_S(\theta_V)$ to $U^S_S(x_S(\theta_V), \theta_S, \psi(\theta_S) = q_S)$. Moreover, since $q_S(\theta)$ increasing in $\theta$, $E$ is also better off, i.e., $U^S_S(x_S(\theta_V), \theta_S, \psi(\theta_S) = q_S) > U^S_S(x_S(\theta_V), \theta_V, \psi(\theta_V) = 0)$. In equilibrium, $E$ will further increase her utility by offering an alternative contract $\tilde{x}_S$ so that $U^S_S(\tilde{x}_S, \theta_S, \psi(\theta_S) = \tilde{q}_S) = U^S_V(\theta_V)$. This further increases $E$’s utility, i.e.,
Indeed, our constraint on the hold-up problem is that the team-incentive problem is not too small (or β not too large), so that \( z_s \leq 1 \).

Consider now mixed financing, in which \( V \) is active and \( S \) passive. In this case the optimal contract maximizes \( W = (P + p_S x_S + p_E x_E)(1 + \beta + \theta) - p_S^2(z_S + \theta) - p_E^2(z_S + \theta) \), subject to the first order conditions \( \sigma_E = p_E(1 + \beta - z_S) \) and \( \sigma_S = p_S(z_S + \theta) \). We immediately see that \( dW/d\theta = (P + p_E^2(1 + \beta - z_S) + p_S^2(1 + \beta + \theta) > 0 \). This shows that \( S \)-financing dominates (is dominated by) \( V \)-financing for \( \theta > 0 \) (\( \theta < 0 \)). It is also useful to note that \( dW/dz_S = (p_S^2 - p_E^2)(1 + \beta + \theta) + p_E^2(1 + \beta - z_S) - p_S^2(\beta_S + \theta) = 0 \), which yields \( z_S = (p_S^2/(p_S^2 + p_E^2))(1 + \beta) - (p_E^2/(p_S^2 + p_E^2))\theta \).

For convenience we assume that \( p_E \) is not too small (or \( \beta \) not too large), so that \( z_S \leq 1 \).

Another interesting aspect to note in this model is that for \( \theta_0 < \theta < 0 \), \( S \) would still want to be involved in the optimal contract. If \( E \) and \( V \) contract on their own, then they would choose \( x_V = (p_S^2/(p_S^2 + p_E^2))(1 + \beta) \). But if they bring in \( S \), then they choose \( x_S^* \) to maximize \( W = (P + p_E^2(1 + \beta - x_V^*) + p_S^2 x_V^*)(1 + \beta + \theta) - p_E^2(1 + \beta - x_V^*)^2/2 - p_S^2(x_V^*)^2/2 - I \), so that \( dW/dx_V = (p_S^2 - p_E^2)(1 + \beta + \theta) + p_E^2(1 + \beta - z_S) - p_S^2(\beta_S + \theta) = 0 \) and thus \( x_V^* = (p_S^2/(p_E^2 + p_S^2))(1 + \beta) + ((p_S^2 - p_E^2)/(p_E^2 + p_S^2))\theta \), which differs from \( x_V \) whenever \( p_E \neq p \). For \( \theta < 0 \), this allocates more shares to \( V \) whenever \( E \) is more productive than \( V \), i.e., \( p_E > p \). This reduces the probability of success. \( S \) obviously needs to make a transfer payment to convince \( E \) and \( V \) to move to this alternative allocation of shares.

Note that we retained the assumption that all returns are paid from profits of the firm, i.e., \( 0 \leq z_i \leq 1 \). If we relaxed this, then it is easy to see that the first-best can always be achieved. All that is required is to set \( z_i = p_i(1 + \beta + \theta) \) for both \( E \) and the active investor. This reflects the well-known result that the team-incentive problem can be solved through budget-breaking. Budget-breaking, however, is generally not collusion proof. Indeed, our constraint \( z_S \geq 0 \) binds naturally if \( E \) and \( V \) could collude to pretend that the venture succeeded, even if it failed.

**References**


