THE NEW ECONOMIC SCHOOL
Macroeconomics 4

Problem Set 2
Year 2002-2003

1 The Time–Averaging Problem

Suppose that consumption follows a random walk: $C_t = C_{t-1} + e_t$, where e is white noise. Suppose, however, that the data provide average consumption over two–period intervals; that is, one observes $(C_t + C_{t+1})/2$, $(C_{t+2} + C_{t+3})/2$, and so on.

(a) Find an expression for the change in measured consumption from one two–period interval to the next in terms of the e's.

(b) Is the change in measured consumption uncorrelated with the previous value of the change in measured consumption? In light of this, is measured consumption a random walk?

(c) Given your result in part (a), is the change in consumption from one two–period interval to the next necessarily uncorrelated with anything known as of the first of these two period intervals? Is it necessarily uncorrelated with anything known as of the two–period interval immediately preceding the first of the two–period intervals?

(d) Suppose that measured consumption for a two–period interval is not the average over the interval, but consumption in the second of two periods. That is, one observes C_{t+1}, C_{t+3}, and so on. In this case, is measured consumption a random walk?

(e) In light of the above results, what problems may emerge when testing the Hall (1978) version of the Permanent Income Hypothesis? What data are best suited for such tests?

2 The Permanent Income Hypothesis and the "Excess Smoothness" Puzzle

Suppose consumption is governed by the permanent income hypothesis, i.e. the consumer maximizes a utility function given by

$$E_t \sum_{s=0}^{\infty} (1+\rho)^{-s} U(C_{t+s})$$

subject to the intertemporal budget constraint

$$E_t \sum_{s=0}^{\infty} (1+r)^{-s} (C_{t+s} - Y_{t+s}) = A_t$$

where:
Y is labor income,
A is non–human wealth, and
r is the (constant and known) real interest rate.

(1) What happens to saving (income less consumption) in response to a positive shock to income ($\varepsilon_t > 0$) if income follows each of the processes below? In answering this question, you may assume that the utility discount rate equals the interest rate ($r = \rho$) and that U(.) is quadratic.

a) $Y_t = \mu + \phi Y_{t-1} + \varepsilon_t$ $(0 < \phi < 1)$

b) $Y_t = Y_{t-1} + \varepsilon_t$

c) $\Delta Y_t = \phi \Delta Y_{t-1} + \varepsilon_t$ $(0 < \phi < 1)$

(2) In light of your results, discuss the following (pseudo) quotation: "The permanent income hypothesis implies that consumers should want to spread out changes in income over the rest of their lives, so consumption should be smoother than income." Your explanation should be clear to a person without any math or economics education.

(3) In U.S. data, consumption is significantly smoother than income, and income seems to follow approximately a random walk (a process like (b) above). Do these findings constitute a puzzle? How might you explain them?

3 The Permanent Income Hypothesis with Durable Goods

Suppose that, as in Section 7.2, the instantaneous utility function is quadratic and the interest rate and the discount rate are zero. Suppose, however, that goods are durable; specifically, $C_t = (1-\delta)C_{t-1} + E_t$, where E_t is purchases in period t and $0 \leq \delta < 1$.

(a) Consider a marginal reduction in purchases in period t of dE_t. Find values of dE_{t+1} and dE_{t+2} such that the combined changes in E_t, E_{t+1}, and E_{t+2} leave the present value of spending unchanged (so $dE_t + dE_{t+1} + dE_{t+2} = 0$) and leave C_{t+2} unchanged (so $(1-\delta)^2 dE_t + (1-\delta)dE_{t+1} + dE_{t+2} = 0$).

(b) What is the effect of the change in part (a) on C_t and C_{t+1}? What is the effect on expected utility?

(c) What condition must C_t and $\mathcal{E}_t[C_{t+1}]$ satisfy for the change in part (a) not to affect expected utility? Does C follow a random walk?

(d) Does E follow a random walk? (Hint: write $E_t - E_{t-1}$ in terms of $C_t - C_{t-1}$ and $C_{t-1} - C_{t-2}$.) Explain intuitively. If $\delta = 0$, what is the behavior of E?

(e) In U.S. quarterly data, all the autocorrelations of $(E_t - E_{t-1})$ are essentially zero. How might you interpret this finding? If you used the explanation implied by part (d), explain why you find that reason plausible.