

**Problem Set 3**  
Year 2002-2003

## 1 Альтернативная спецификация издержек освоения капитала

Представьте фирму, которая инвестирует в капитал и отдает этот капитал в аренду по экзогенно заданной постоянной ставке  $R$  (частичное равновесие). Таким образом, максимизационная задача фирмы выглядит так:

$$\max_I \int_0^{\infty} e^{-rt} [RK_t - I_t] dt.$$

при динамическом условии

$$\dot{K} = KJ(I/K), J' > 0, J'' < 0, J(\delta) = 0, J'(\delta) = 1$$

где  $\delta$  - норма амортизации.

а) Дайте четкую интерпретацию динамическому условию для капитала, включая все четыре условия на функцию  $J(I/K)$ .

б) Выведите условия первого порядка для этой задачи. Насколько они согласуются с выведенными в классе?

в) Существует ли стационарный режим в данной модели, где стационарный режим определяется как  $\dot{K} = \dot{q} = 0$ ? Если да, то при каких условиях, и является ли он единственным? Дайте четкую интуицию, почему при других условиях стационарный режим не существует.

г) Изобразите получившуюся систему на фазовом портрете в пространстве  $q, K$ , с капиталом, отложенном по горизонтальной оси, для случая, когда стационарный режим существует.

## 2 Investment and the Housing Market

Consider the following model of the housing market (due to Jim Poterba):

$$\begin{aligned} I &= \psi(P), \quad \psi' > 0 \\ r + \delta &= \frac{R + \dot{P}}{P} \\ R &= R(H). \quad R' < 0 \\ \dot{H} &= I - \delta H \end{aligned}$$

where  $I$  is gross investment in housing,  $P$  is the price of a house,  $R$  is the rental cost of a house, and  $r$  is the real interest rate.

(a) Explain why each of the equations of the model is reasonable.

(b) Write the model in terms of two variables (a state variable and costate variable) and two equations of motion.

(c) In a phase diagram, display the system's dynamics

- (d) What is the steady state effect on  $H, P, I$ , and  $R$  of an increase in the real interest rate  $r$ ?
- (e) What is the effect on  $h, P, I$ , and  $R$  over time of an unanticipated, permanent increase in the rental rate  $r$ ?
- (f) What is the effect of an unanticipated, temporary increase in real interest rate?
- (g) What is the effect of a pre-announced future permanent increase in the real interest rate?
- (h) Suppose that instead of having rational expectations (here, perfect foresight) about the price of a house, people have static expectations - they expect that the price of a house will never change from what it is now. Redo part (e) under this new assumption

### 3 Asymmetric Information and Market Collapse

Consider the following model of student loans (following Gregory N. Mankiw).

Each student decides whether to take a bank loan and invest in education, which will bring an expected return  $R$  to the student, but with probability  $P$  the student will default and pay nothing to the bank.  $R, P$  are heterogeneous across students; students know their own values, but the bank only knows the general distributions. Specifically, the average probability of default is  $\Pi$ . The bank can invest into a risk-free security with return  $\rho$  or lend to student at interest rate  $r$ . The banks and students are risk-neutral.

- (a) Equalizing returns for banks, what is the relation between  $r$  and  $\rho$  given that the banking sector is competitive? Call this condition (1).
- (b) For the student, what is the expected return and expected payment for loan? Under what condition will the student take a loan? Draw this condition in  $P, R$  space. Will all students for whom  $R > \rho$  take the loan? What about the other? Why or why not?
- (c) For the bank, what is the probability of repayment conditional on the knowledge about what type of students take the loan (use the answer to (b) in answering this question)? Write down the corresponding function  $\Pi(r)$ , and call it condition (2).
- (d) Draw conditions (1) and (2) in the  $\Pi, r$  space. Will there always be an equilibrium?
- (e) Solve for a particular case when  $R$  is constant and  $P$  is uniform. Under what conditions will there be an equilibrium?
- (f) What does this model imply about the sensitivity of investment with respect to the interest rate? Can this model help understand the collapse of investment during the Great Depression in the U.S. and in Russia of 1990s?