THE NEW ECONOMIC SCHOOL
Macroeconomics 4

Problem Set 4
Year 2002-2003

1. The Lucas asset pricing model

Consider an economy consisting of many identical immortal consumers, who maximize

$$E_t \sum_{s=0}^{\infty} (1+\rho)^{-s} \ln(C_{t+s})$$

with standard notation. The consumption good in this economy is a non-storable fruit, which is produced by immortal and identical trees. For simplicity, assume the number of trees equal the number of consumers. Fruit production by any one tree, Y_t, is random; it is known at time t, but not at time $t-1$. Let P_t denote the price of a tree.

(a) Every period, each consumer must decide whether to forgo some consumption in order to add another tree to her portfolio. Write down the first-order condition characterizing the representative consumer's optimal program.

(b) In equilibrium, $C_t = Y_t$. Why? Is consumption a random walk? Why or why not?

(c) Prove that the equilibrium price of a tree is

$$P_t = \frac{Y_t}{\rho}.$$

(d) Suppose the news arrives (e.g. good weather) that increases the expected amount of fruit the trees will produce in the future. What happens to the price of a tree today? What happens to consumption today? Give both a formal and intuitive explanations.

(e) What is a consumer's first-order condition with respect to an asset with risk-free return r^f? Use this condition to solve for the equilibrium risk-free interest rate in this economy. Given that the only asset that exists (the tree) is risky, what is the meaning of this equilibrium risk-free interest rate?

2. Stock market volatility

Solve problems 1 and 4 in Chapter 5, Blanchard and Fischer (pages 266-267).

(c) Briefly describe the Shiller test and discuss how the assumption on the value of ρ affects its validity.