MEM or/and logARMA: which model for realized volatility?

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How to model RV

Realized volatility (RV): Andersen, Bollerslev, Diebold, and Labys (2003)

- *LogARMA models* – ARMA applied to logarithm of RV
  - VAR-RV in Andersen, Bollerslev, Diebold, and Labys (2003)
  - Somewhat motivated by approximate log-normality of RV distribution
  - Familiar class of models, clear how to work with
  - Allows familiar extensions: GARCH errors, non-normal error distributions, fancier mean dynamics (e.g., HAR of Corsi, 2009)
- *MEM models* – Multiplicative error models (Engle, 2002)
  - RV is positive variable like, e.g. trade durations
  - Autoregressive conditional durations (ACD) models for intradaily trade durations from Engle & Russell (1998)
  - New class of models, less clear how to work with
  - Harder to extend: breaking dynamics of mean and variance, non-exponential distributions (e.g., Hautsch, 2011), copulas for multivariate analysis (e.g., Engle and Gallo, 2006)

Objective of this study

- Confront MEM and LogARMA model classes for RV
- Specify 2 members for each class – 'small' model and 'big' model
  - 'Small model' uses baseline QML conditional distribution
  - 'Big model' uses sophisticated conditional distribution with additional shape parameters
  - + 2 model averages over these 4 'extreme' models
- Using real data on realized volatility, try to determine the 'optimal' modeling strategy
- Compare in-sample and out-of-sample performance, paying special attention to forecasting quality

MEM: equations

$$rv_t = \mu_t \varepsilon_t$$

where $\mu_t = E_{t-1} rv_t$ and $\varepsilon_t > 0$ with $E[\varepsilon_t | I_{t-1}] = 1$

$$\log \mu_t = \omega + \alpha \log rv_{t-1} + \beta \log \mu_{t-1}$$

SMALL MEM: *ExpMEM* model

$$\varepsilon_t | I_{t-1} \sim \mathcal{E}$$

where $\mathcal{E}$ denotes standard exponential distribution

BIG MEM: *BurrMEM* model

$$\varepsilon_t | I_{t-1} \sim \mathcal{B}(\zeta, \varrho)$$

where $\mathcal{B}(\zeta, \varrho)$ is standardized Burr distribution with parameters ζ and ϱ

MEM: predictions

For both ExpMEM and BurrMEM

$$\widehat{rv}_{t+1} = \exp\left(\hat{\omega} + \hat{\alpha}\log rv_t + \hat{\beta}\log\hat{\mu}_t\right)$$

LogARMA: equations

$$\log rv_t = \mu + \phi \log rv_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$$

where $\varepsilon_t = \sigma_t \eta_t$ with EGARCH dynamics for σ_t and $\eta_t | I_{t-1} \sim (0, 1)$

SMALL LogARMA: *NormLogARMA* model

$$\eta_t | I_{t-1} \sim \mathcal{N}$$

where $\mathcal{N}$ denotes standard normal distribution

BIG LogARMA: *SkStLogARMA* model

$$\eta_t | I_{t-1} \sim \mathcal{SS}(\lambda, \nu)$$

where $\mathcal{SS}(\lambda, \nu)$ denotes standardized Skewed Student distribution with parameters λ (for asymmetry) and ν (for tail thickness)

LogARMA: predictions

For both NormLogARMA and SkStLogARMA mean prediction is

$$\widehat{\log rv}_{t+1} = \hat{\mu} + \hat{\phi} \log rv_t + \hat{\theta}\varepsilon_t$$

and volatility prediction $\hat{\sigma}^2_{t+1}$ from EGARCH

For the NormLogARMA model:

$$\widehat{rv}_{t+1} = \exp\left(\widehat{\log rv}_{t+1} + \frac{1}{2}\hat{\sigma}^2_{t+1}\right)$$

For the SkStLogARMA model, $\mathbb{E}[rv_{t+1}|I_t] = \mathbb{E}[\exp(\sigma_{t+1}\eta_{t+1})|I_t]$ does not have closed form, therefore

$$\widehat{rv}_{t+1} = \exp\left(\widehat{\log rv}_{t+1}\right)\widehat{\mathbb{E}}[\exp(\sigma_{t+1}\eta_{t+1})|I_t]$$

where $\widehat{\mathbb{E}}[\exp(\sigma_{t+1}\eta_{t+1})|I_t] = \int_{-\infty}^{+\infty} \exp(\widehat{\sigma}_{t+1}\eta) f_{\mathcal{SS}}(\eta, \hat{\lambda}, \hat{\nu}) d\eta$ is computed numerically using Gauss-Chebychev quadrature

Model averaging, in-sample based

Combined forecasts based on weights from in-sample information criteria (Buckland, Burnham & Augustin, 1997)

Smoothed TIC (STIC) predictions:

$$\widehat{rv}_{t+1} = \sum_{i=1}^{M} w_i \widehat{rv}_{t+1,i} \qquad w_i = \frac{\exp(-TIC_i/2)}{\sum_{j=1}^{M} \exp(-TIC_j/2)} \quad i = 1, \dots, M$$

In-sample TIC for each model (Takeuchi, 1976)

$$TIC = -2\ell_n(\hat{\theta}) + 2\mathrm{tr}\big(\hat{J}^{-1}\hat{I}\big)$$

where $\ell_n(\hat{\theta}) = \sum_t \log f_{t-1}(rv_t|\theta)$ is loglikelihood function
$\hat{J}$ and $\hat{I}$ are estimates of $J = E\left[\partial^2 \log f_{t-1}(rv_t|\theta)/\partial\theta\partial\theta'\right]$
and $I = E\left[\partial \log f_{t-1}(rv_t|\theta)/\partial\theta\, \partial \log f_{t-1}(rv_t|\theta)/\partial\theta'\right]$

(correct specification $\Rightarrow$ $J = I$ $\Rightarrow$ $\mathrm{tr}\left(J^{-1}I\right) = \dim(\theta)$, and TIC = AIC)

Model averaging, out-of-sample based

Combined forecasts based on out-of-sample performance of individual models

Jackknife model averaging (JMA) machinery (Hansen & Racine, 2012):

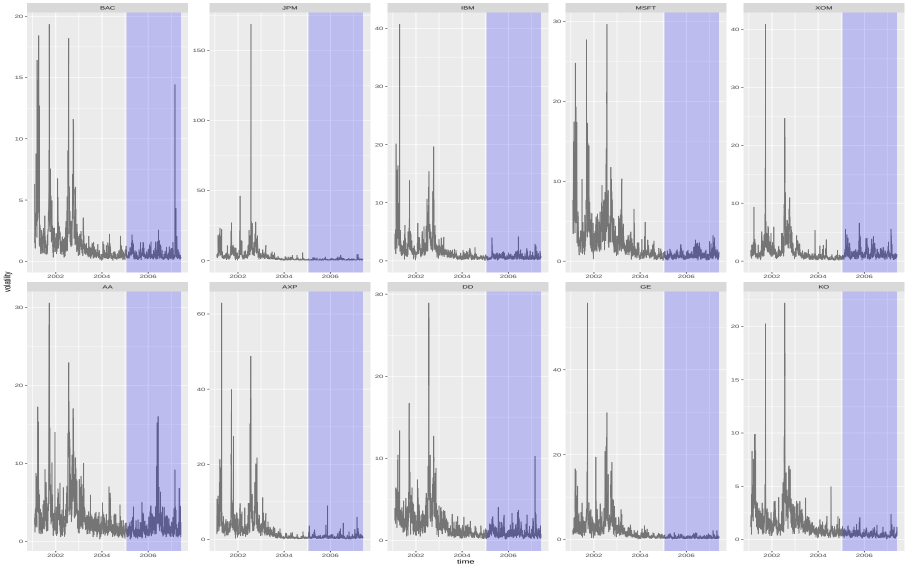
weights w_i, $i = 1, \ldots, M$, are determined by minimizing CV criterion computed from M sets of forecast errors on validation subsample

Experimental setting

We adopt RV data from Noureldin et al (JAE 2012) also used elsewhere

- Period: 2/2001–05/2007
- 10 stocks: BAC, JPM, IBM, MSFT, XOM, AA, AXP, DD, GE, KO
- 1589 observations in total, 589 for forecast evaluation, estimation in 1000-observations rolling window
- 6 sets of predictions:
  - from ExpMEM (small MEM model)
  - from BurrMEM (big MEM model)
  - from NormLogARMA (small LogARMA model)
  - from SkStLogARMA (big LogARMA model)
  - from STICMA
  - from JMA
- 1-step ahead forecasting performance measured by MSPE and model confidence sets (MSC, Hansen, Lunde & Nason, 2011) over MSPE

Data on realized volatilities



Model estimates for BAC in first window: MEM

ExpMEM model

$$\varepsilon_t | I_{t-1} \sim \mathcal{E}$$

$$\log \mu_t = \underset{(0.0096)}{0.0420} + \underset{(0.193)}{0.363} \log rv_{t-1} + \underset{(0.215)}{0.615} \log \mu_{t-1}$$

$$LL = -1165.7, \quad TIC = 2333.8$$

BurrMEM model

$$\varepsilon_t | I_{t-1} \sim \mathcal{B}\left(\underset{(0.159)}{3.081}, \underset{(0.198)}{1.277} \right)$$

$$\log \mu_t = \underset{(0.0117)}{0.0438} + \underset{(0.140)}{0.371} \log rv_{t-1} + \underset{(0.150)}{0.611} \log \mu_{t-1}$$

$$LL = -675.0, \quad TIC = 1361.2$$

Model estimates for BAC in first window: LogARMA

NormLogARMA model

$$\log rv_t = \underset{(0.00569)}{-0.00267} + \underset{(0.009)}{0.977} \log rv_{t-i} + \varepsilon_t - \underset{(0.040)}{0.611} \varepsilon_{t-j}$$

$$\log \sigma_t^2 = \underset{(2.375)}{-0.155} + \underset{(0.275)}{0.071} |\eta_{t-1}| - \underset{(0.021)}{0.013} \eta_{t-1} + \underset{(0.979)}{0.938} \log \sigma_{t-1}^2$$

$$\eta_t | I_{t-1} \sim \mathcal{N}$$

$$LL = -625.6, \quad TIC = 1383.4$$

SkStLogARMA model

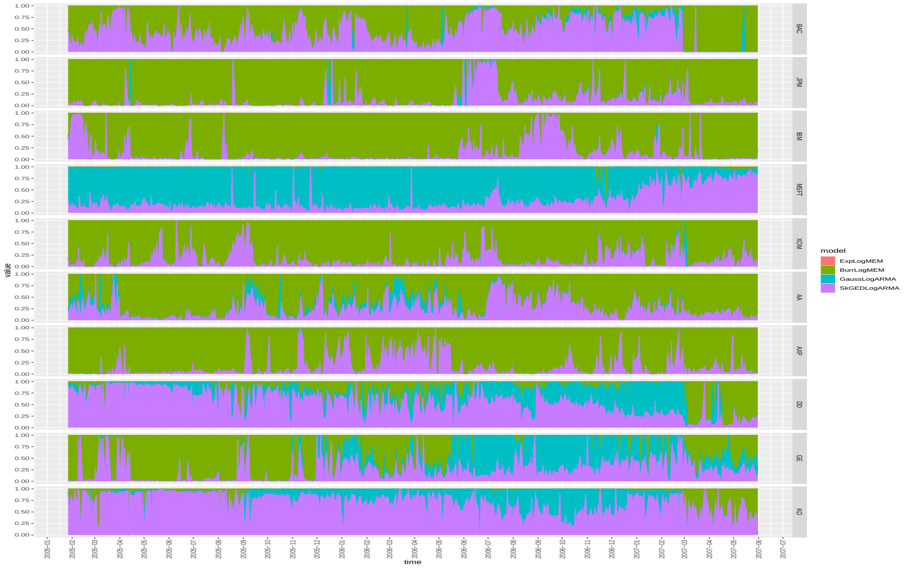
$$\log rv_t = \underset{(0.00591)}{-0.00077} + \underset{(0.009)}{0.981} \log rv_{t-i} + \varepsilon_t - \underset{(0.033)}{0.611} \varepsilon_{t-j}$$

$$\log \sigma_t^2 = -\underset{(10.231)}{0.809} + \underset{(0.179)}{0.112} |\eta_{t-1}| - \underset{(0.046)}{0.000} \eta_{t-1} + \underset{(5.810)}{0.545} \log \sigma_{t-1}^2$$

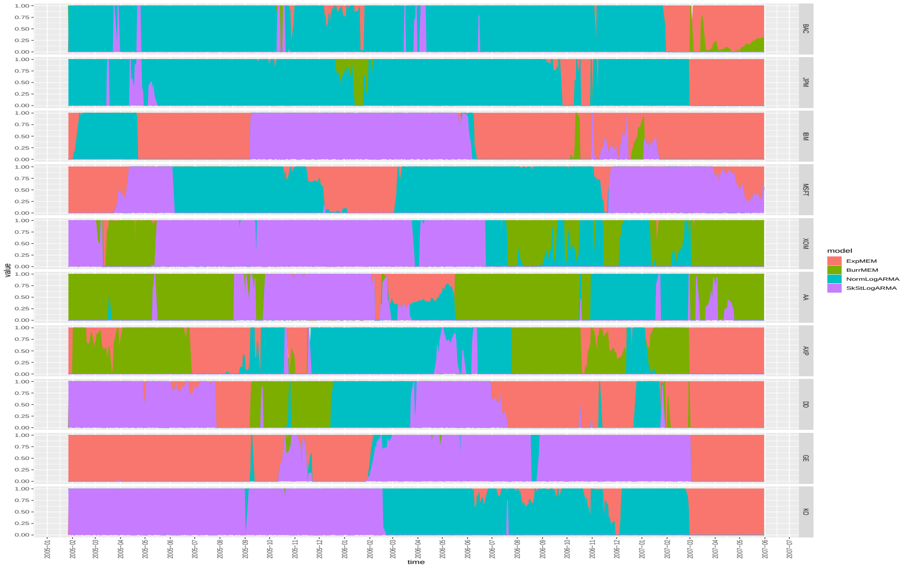
$$\eta_t | I_{t-1} \sim \mathcal{SS}\left(\underset{(0.0370)}{0.0927}, \underset{(0.102)}{1.546}\right)$$

$$LL = -615.3, \quad TIC = 1361.4$$

STICMA weights in rolling window



JMA weights in rolling window



Predictive model confidence sets for 25% confidence level

	BAC	JPM	IBM	MSFT	XOM	GE	KO
ExpMEM	*					*	
BurrMEM					*		
NormLogARMA	*	*	*	*	*	*	*
SkStLogARMA		*	*		*	*	*
STICMA				*	*		*
JMA	*	*	*		*	*	*

(skipping AA AXP DD whose MCS contain all models)

Summary of findings

- In-sample, ExpMEM is no match to others
- In forecasting, LogARMA class is more reliable than MEM class
- NormLogARMA seems most reliable in LogARMA class
- Each model appears at times superior (short-lived)
- For different assets, different models are superior
- Superiority varies in time
- Model averaging: one, at most two models are used simultaneously
- Model averaging: JMA barely improves forecasting, STICMA doesn't