New Economic School
Contract Theory
Final Exam

Sergei Guriev, Andrei Bremzen
T/A: Andrew Vashevnik, Anton Kolotilin

Monday, October 23, 2006.

This is an open book exam. Please answer all questions below. You have 3 hours. Good luck!

1. As in Hart's chapter 2, assume that there are two managers $M1$ and $M2$ operating respectively assets $a1$ and $a2$. The second manager, working with the second asset, produces one unit of input for the first manager working on the first asset. Ex ante uncertainty about the relevant specification of the input makes explicit long term contracts impossible. $M1$ (resp. $M2$) makes relationship-specific investment i (resp. e) at costs $\frac{i^2}{2}$ (resp., $\frac{e^2}{2}$) – **mind quadratic costs**; both i and e are ex post observable but not verifiable. Once investments are sunk, the relevant type of the input becomes contractible and the parties negotiate the price of the input; they have equal bargaining power during the negotiations (which proceeds à la Rubinstein). Generic inputs are traded on the (competitive) market at price $\overline{p} = 100$, but using them is less attractive for $M1$ and producing them is more costly for $M2$.

   Given investment level i, manager $M1$ (if supplied the input) can produce the final output worth $R(i) = 100+8i$. With generic input his output is worth $r(i, \{a1, a2\}) = 100+6i$ if he owns both assets, $r(i, \{a1\}) = 10 + 2i$ if he only owns $a1$ and $r(i, \emptyset) = 100$ if he owns none of the assets (or owns a2). Likewise, given investment level e, the costs for the second manager of producing relationship-specific input are $C(e) = 100 - 8e$, while those of producing generic input, dependent on who owns the assets, are $c(e, \{a1, a2\}) = 100 - 6e$, $c(e, \{a2\}) = 100 - 2e$ and $c(e, \emptyset) = 100$.

   (a) Find first best level of investments i^* and e^* and first best welfare.

   (b) Under each of the three ownership structures find equilibrium levels of investments and welfare. Determine the optimal ownership structure.

   Now assume that these two managers interact each period $t = 0, 1, \ldots$. Each period they first determine the ownership structure, than choose noncontractible investments, then bargain over the output price. There is no discounting within each period, but the interest rate between two subsequent periods is $r > 0$. Managers ex ante (informally) agree on i and e; if either of them chooses the level of investments other that the one they agreed on, the other manager no longer trusts her and they draw back to their noncooperative level of invesments (that you calculated in (b)) in all future periods.

   (c) Assume that each manager owns her respective asset and that it is inalienable (i.e., for a legal reason you can not sell the asset you are working on). Find the maximum level of r for which i^* and e^* are achievable. Find the maximum achievable level of wealth as a function of r.

   (d) Redo (c) but now assume that assets can be freely traded before each period. Is it now easier or harder for the managers to achieve implicit agreement to invest i^* and e^*?

2. Principal P hires an agent A. Agent chooses effort $a \geq 0$ that affects P's payoff $x = a + \varepsilon$ where ε is a normal random variable with zero mean and variance σ^2. Agent has CARA utility and quadratic cost of effort $\theta a^2/2$ so she maximizes $\mathrm{E}\left[1 - e^{-r(w-\theta a^2/2)}\right]$. Her reservation utility is $1 - e^{-ru_0}$. The principal offers the agent a linear contract $w(x) = \alpha x + \beta$. P is risk-neutral and maximizes $\mathrm{E}[x - w]$. A's type θ can either be $\underline{\theta}$ (with probability λ) or $\overline{\theta}$ (with probability $1 - \lambda$).

   (a) Find the first best level of effort for each type. Calculate P's and A's payoffs.

   (b) Suppose that θ is observed. Find the optimal linear contract for each type. Compare α and β for the two types.

   (c) Now assume that θ is A's private information. Solve for the optimal menu of linear contracts $\{\underline{\alpha}, \underline{\beta}, \overline{\alpha}, \overline{\beta}\}$.

   (d) What would change in (c) if the reservation utility u_0 were different for the high and the low type (consider the cases $\underline{u_0} < \overline{u_0}$ and $\underline{u_0} > \overline{u_0}$).

3. A principal P has two agents $i = 1, 2$. Each agent i exerts effort along two dimensions x_i and y_i. The cost of effort is $x_i^2/2 + y_i^2/2 + kx_iy_i$ where $k \in (-1, 1)$. Principal's payoff is $\mathrm{E}[X + Y]$ where X, Y are independent binary random variables taking values 0 and 1. The probability of $X = 1$ is $x_1 + \delta x_2$, the probability of $Y = 1$ is $y_2 + \delta y_1$; here $\delta < 1$. The agents have limited liability so the principal offers each agent a contract $\{\xi_i, \eta_i\}$: agent i gets paid $\xi_i X + \eta_i Y$.

   (a) Find the first best effort levels.

   (b) Given the contracts $\{\xi_i, \eta_i\}$ find the effort choices of the agents.

   (c) Solve for the optimal contract for the principal. Compare to (a). Carry out comparative statics with regard to k and δ. Provide intuition.

   (d) Redo (c) in the case where P's utility is $\mathrm{E}[XY]$.

4. There are two identical assets that belong to the principal. There are three tasks associated with the assets: two involve specialization on (or 'thinking about') each of the assets separately and the third involves coordinating the use of both. The principal may hire agents (there are many available, each with reservation wage zero and no liquidity constraints, so they can pay upfront for a position) to work for her on either task. The principal and all agents are risk neutral and identical in terms of productivity on any task.

   If thinking about a single asset is successful (the idea is produced) it can generate private value $v(\{a_1\}) = v(\{a_2\}) = 5$, if thinking about the pair of assets is productive it can generate value $V \leq 20$. The value of a task is generated only if the idea gets to be implemented, which only happens if the agent who works on this task has access to all assets in the task (i.e., if no agent superior to him on any asset has an idea).

   Thinking about a task generates an idea with probability that is a function of effort. An agent can set this probability at p_s at private costs $\frac{10p_s^2}{2}$ for a task involving just one asset and at p_c at costs $\frac{20p_c^2}{2}$ for a task involving both assets.

   The timing is as follows: the principal moves first by designing a complete hierarchy, i.e., deciding how many agents to hire and how to allocate the chain of command over each asset and then making take it or leave it offers to all agents. Each potential employee decides whether to accept the offer (consisting of a position in the chain of command and a transfer to be paid upfront); those who accepted then choose their efforts. Then ideas are (stochastically) generated; those agents whose ideas get to be implemented receive their private benefits.

   (a) Assume that there are just two agents available on the labor market. What hierarchy will the principal choose? What will be the total welfare? (Your answer would depend on V).

   (b) Redo (a) assuming that there are three agents available on the market.

   (c) For this point only assume $V = 15$. Assume now that there are potentially many agents but there are positive hiring costs of 1 per agent. How many agents will the principal hire? What hierarchy will she design?