Online Appendix
Uncovering the skewness news impact curve

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A Robustness of skewness NIC for S&P500

Our analysis is parametric, and therefore the conclusions made may be prone to misspecification of various functional forms. Therefore, to confirm that the shape of the obtained skewness NIC is robust to various deviations in functional forms, we run a number of robustness checks. These experiments can be divided into four groups: change in the data span, change in specification of the mean equation, change in specification of the variance equation, and change in specification of the density function. Table 1 shows the estimates of the 'transition' specification of the skewness equation in each of these experiments applied to the S&P500 index. The column 'main' replicates the estimates from Table 2 from the main text.

Table 1: Results of robustness checks for S&P500 index

	S&P500				
variation →	main	1990–	+AR(1)	$+\sigma_t^2$-M	(1,2)
κ_0	−0.050 (0.016)	−0.077 (0.032)	−0.050 (0.134)	−0.048 (0.022)	−0.050 (0.016)
κ_{-1}	0.606 (0.088)	0.551 (0.152)	0.571 (0.067)	0.607 (0.104)	0.592 (0.059)
κ_3	0.231 (0.042)	0.284 (0.063)	0.273 (0.026)	0.236 (0.035)	0.251 (0.036)
ν_λ	−0.212 (0.021)	−0.280 (0.099)	−0.206 (0.031)	−0.217 (0.012)	−0.218 (0.023)
variation →	HGARCH	CGARCH	+Jumps	Skew-t	G–C
κ_0	−0.049 (0.016)	−0.033 (0.017)	−0.032 (0.019)	−0.036 (0.014)	−0.109 (0.043)
κ_{-1}	0.620 (0.079)	0.573 (0.079)	0.634 (0.100)	0.624 (0.077)	0.701 (0.068)
κ_3	0.235 (0.047)	0.228 (0.049)	0.200 (0.039)	0.198 (0.038)	0.460 (0.131)
ν_λ	−0.210 (0.027)	−0.199 (0.066)	−0.208 (0.029)	−0.206 (0.040)	−0.080 (0.066)

Notes: Robust standard errors are in parentheses. Model variation labels are decrypted in text.

Our dataset includes the observation made on October 19, 1987, the day known as 'Black Monday.' On this day, the S&P500 index experienced the largest drop in history during one trading day: it lost more than 20%. We check whether the exclusion of the decade of the 80's with its turbulent period in October 1987 alters the skewness NIC estimates. The skewness parameters shown in the column '1990–' of Table 1 change a little, but the qualitative properties of the skewness NIC do not change. A more noticeable change occurs in the standard errors (to a greater extent than the change in the sample size implies), especially to that of ν_λ; evidently, the October 1987 extremal returns are quite informative about this coefficient.

Recall that our specification for the conditional mean is trivial, a constant. We verify that accounting for possible (weak) serial correlation may change estimates of the skewness NIC. We use two popular specifications: an additional autoregressive term ρr_{t-1} and an additional GARCH-M term $\gamma\sigma_t^2$. The columns '+AR(1)' and '+σ_t^2-M' in Table 1 show that the skewness NIC parameters change very little as a result, with the skewness NIC keeping its previous shape.

We also estimate the model with various perturbations of the volatility equation. This is important because misspecification of conditional variance is likely to pass over to the next conditional moment, i.e. conditional skewness. First we check whether increasing the order of the EGARCH model by adding the standardized returns and their absolute values lagged twice is able to dampen the shape of the skewness NIC. The column '(1,2)' indicates that it does not. Next, we replace the EGARCH process for the conditional variance by alternative volatility specifications. A flexible specification of Hentschel (1995) is

$$\sigma_t^{2\theta} = \beta_0 + \beta_{-1}\sigma_{t-1}^{2\theta} + \beta_1\sigma_{t-1}^{2\theta}(|z_{t-1} - b| - c(z_{t-1} - b))^{\zeta_\sigma},$$

where $\theta > 0$, $\zeta_\sigma > 0$. This specification allows many forms of leverage and includes standard GARCH, EGARCH, GJR–GARCH, Threshold GARCH, Asymmetric Power GARCH and others as special cases. Another GARCH model, Component GARCH (Engle & Lee, 1999), models both short-run and long-run movements in volatility. The short-run movements around the long-run component q_t follow

$$\sigma_t^2 = q_t + \beta_{-1}(\sigma_{t-1}^2 - q_{t-1}) + \beta_1(\sigma_{t-1}^2 z_{t-1}^2 - q_{t-1}),$$

where q_t follows

$$q_t = \gamma_0 + \gamma_{-1}q_{t-1} + \gamma_1\sigma_{t-1}^2(z_{t-1}^2 - 1).$$

This volatility specification may also be represented as GARCH(2,2) with certain equality constraints placed on parameters. The columns 'HGARCH' and 'CGARCH' in Table 1 correspond to these two extended GARCH models and show that the parameters of skewness NIC change very little after such replacements, if at all.

Next, we amend the SGE density specification with the possibility of jumps as in Jorion (1988). The idea is that if extreme shocks are really responsible for the pattern found through the skewness NIC, their explicit introduction into the model should make the skewness NIC differently shaped. However, as shown in the column '+Jumps', the incorporation of jumps, while increasing the likelihood value (not shown) by a non-negligible amount, changes the skewness coefficient little, and does not question the shape of the skewness NIC.

In the last set of robustness experiments, we replace the SGE density with that of Skewed-t and Gram–Charlier distributions. The Skewed-t distribution is another workhorse in estimating skewed conditional distributions in empirical finance; see, for example, Hansen (1994) and Jondeau & Rockinger (2003). It has two parameters: the asymmetry parameter λ and tail thickness parameter (degrees of freedom) ν. Standardized to have zero mean and unit variance, it has the following probability density function:

$$f(z, s, k) = bc\left(1 + \frac{\xi^2}{\nu - 2}\right)^{-(\nu+1)/2}, \tag{1}$$

where $\xi = (bz + a)/(1 - \lambda)$ if $z < -a/b$ and $\xi = (bz + a)/(1 + \lambda)$ otherwise, $c = \Gamma((\nu + 1)/2)/\Gamma(\nu/2)/\sqrt{\pi(\nu - 2)}$, $a = 4c\lambda(\nu - 2)/(\nu - 1)$, and $b = \sqrt{1 + 3\lambda^2 - a^2}$. We again parameterize the asymmetry parameter λ via the same logistic transformation as for the SGE density and the 'transition' specification for the underlying process.

The Gram–Charlier distribution has also gained popularity in empirical finance; see, for example, Jondeau & Rockinger (2001) and León et al. (2005). It has two parameters: the skewness s and kurtosis k. Standardized to have zero mean and unit variance, it has the following probability density function:

$$f(z, s, k) = \phi(z)\left(1 + \frac{s}{3!}(z^3 - 3z) + \frac{k - 3}{4!}(z^4 - 6z^2 + 3)\right), \tag{2}$$

where $\phi(z)$ is the probability density of the standard normal distribution, and s and k are parameters of the distribution associated with skewness and kurtosis, respectively:

$$\int_{-\infty}^{\infty} z^3 f(z, s, k)dz = s, \quad \int_{-\infty}^{\infty} z^4 f(z, s, k)dz = k.$$

We keep the kurtosis parameter k constant while setting the dynamics of the parameter s through another logistic transformation

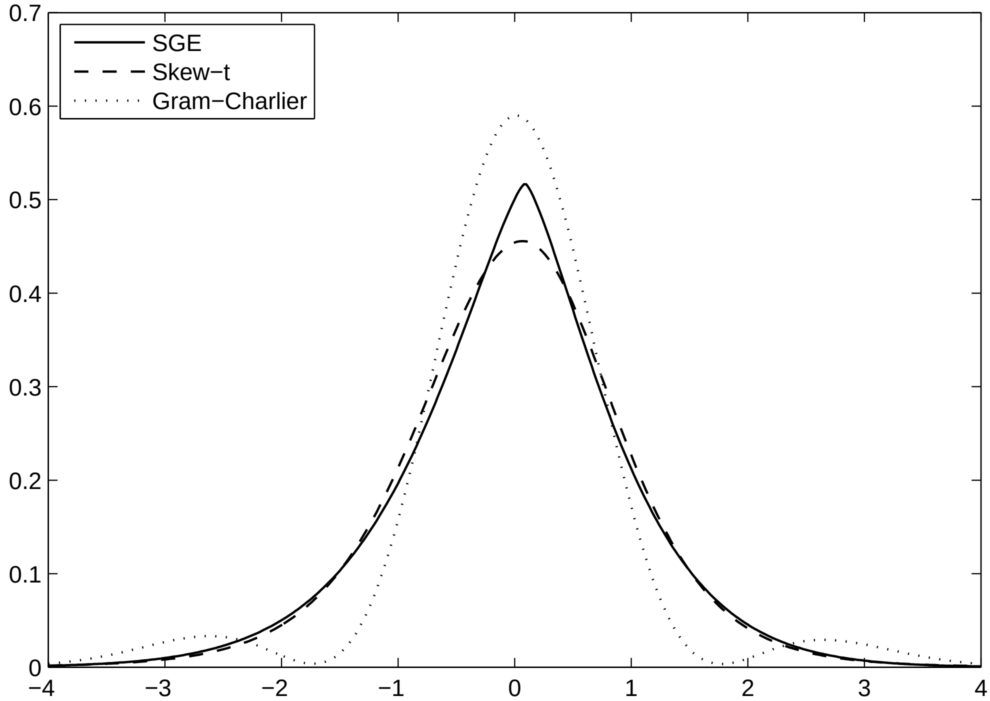
$$s_t = s_{min}(k) + \frac{s_{max}(k) - s_{min}(k)}{1 + \exp(-\tilde{\lambda}_t)},$$

where $s_{min}(k)$ and $s_{min}(k)$ are minimal and maximal possible values of the parameter s_t for given k, for which the Gram–Charlier density is defined;[1] as before, $\tilde{\lambda}_t$ follows the

[1]The expression (2) represents a proper density function only for a limited set of values of parameters s and k, while for other values, the function $f(\cdot, s, k)$ in (2) may not be positive. Jondeau & Rockinger (2001) find the region for s and k within which expression (2) is positive for any real z. This fact should be taken into account while modeling the skewness dynamics using the Gram–Charlier distribution.

dynamics of the 'transition' specification.

Figure 1: Skewed GED, Skewed-t and Gram–Charlier densities with estimated shape parameters



The columns 'Skew-t' and 'G–C' in Table 1 correspond to these two alternative densities. For the Skewed-t case, the estimates of skewness parameters differ just a bit from those in the column 'main' and are practically equal to those in the column '+Jumps', while the likelihood value (not shown) is very close. The non-monotone shape for the skewness NIC is in place. For the Gram–Charlier case, the skewness coefficients are much farther away. However, the likelihood value (not shown), which is quite a bit lower, indicates that this distribution is less suitable for adequately modeling a return distribution with non-trivial dynamics for higher order moments and values thereof that are sizably different from those of the normal distribution.[2] Figure 1 graphs the three densities for (averaged when time varying) estimated values of shape parameters. One can see that the SGE and Skewed-t densities are pretty close, while the Gram–Charlier density puts too much probability mass on small and large returns at the expense of medium-sized returns.

[2]Recall that the Gram–Charlier density is derived from an expansion around the normal density.

B Direct models for skewness

The model of León et al. (2005) reads:

$$r_t = \alpha r_{t-1} + \sigma_t z_t,$$

where $\mathbb{E}z_t = 0$ and $\mathbb{V}z_t = 1$. The conditional variance σ_t^2 follows nonlinear asymmetric GARCH dynamics:

$$\sigma_t^2 = \beta_0 + \beta_{-1}\sigma_{t-1}^2 + \beta_1\sigma_{t-1}^2 \left(\beta_\nabla + z_{t-1}\right)^2,$$

and the conditional skewness s_t and kurtosis k_t follow the GARCH-like dynamics:

$$\begin{aligned} s_t &= \kappa_0 + \kappa_{-1}s_{t-1} + \kappa_1 z_{t-1}^3, \\ k_t &= \delta_0 + \delta_{-1}k_{t-1} + \delta_1 z_{t-1}^4. \end{aligned}$$

The conditional density of r_t is

$$f_{t-1}(r_t) = \sqrt{\frac{1}{2\pi}\frac{\psi_t^2}{\sigma_t\chi_t}}e^{-\frac{1}{2}z_t^2},$$

where

$$\begin{aligned} \psi_t &= 1 + \frac{1}{6}s_t(z_t^3 - 3z_t) + \frac{1}{24}(k_t - 3)(z_t^4 - 6z_t^2 + 3), \\ \chi_t &= 1 + \frac{1}{6}s_t^2 + \frac{1}{24}(k_t - 3)^2. \end{aligned}$$

It is straightforward to verify that in the León et al. (2005) model s_t ceases to be conditional skewness (and k_t to be conditional kurtosis) after the Gram–Charlier polynomial ψ_t is squared to ensure positivity of the density.

The model of Harvey & Siddique (1999) reads

$$r_t = c + m_t + \sigma_t z_t,$$

where $\mathbb{E}z_t = 0$ and $\mathbb{V}z_t = 1$. Here m_t is the GARCH-M term:

$$m_t = \zeta\sigma_t^2,$$

the conditional variance σ_{t+1}^2 follows the GARCH dynamics:

$$\sigma_t^2 = \beta_0 + \beta_{-1}\sigma_{t-1}^2 + \beta_1\sigma_{t-1}^2 z_{t-1}^2,$$

and the conditional (central) skewness s_t follows the GARCH-like dynamics:

$$s_t = \kappa_0 + \kappa_{-1} s_{t-1} + \kappa_1 \sigma_{t-1}^3 z_{t-1}^3.$$

Let $\mu_{1,t}$, $\mu_{2,t}$ and $\mu_{3,t}$ denote the 1^{st}, 2^{nd} and 3^{rd} central moments of the non-central t distribution with parameters ν_t and δ_t. These moments are related to the conditional moments of r_t via

$$\frac{m_t}{\sigma_t} = \frac{\mu_{1,t}}{\sqrt{\mu_{2,t}}}, \quad \frac{s_t}{\sigma_t^3} = \frac{\mu_{3,t}}{\mu_{2,t}^{3/2}}.$$

Denote

$$\xi_t = \Gamma\left(\frac{\nu_t}{2}\right)^{-1} \Gamma\left(\frac{\nu_t - 1}{2}\right) \sqrt{\frac{\nu_t}{2}}.$$

The mapping between the 1^{st} and 3^{rd} central moments and distribution parameters is

$$\mu_{1,t} = \xi_t \delta_t$$

and[3]

$$\mu_{3,t} = \left(\frac{\nu_t(7 - 2\nu_t)}{(\nu_t - 2)(\nu_t - 3)} + 2\xi_t^2\right) \xi_t \delta_t^3 + \frac{3\nu_t}{(\nu_t - 2)(\nu_t - 3)} \xi_t \delta_t,$$

while the 2^{nd} central moment (variance) equals

$$\mu_{2,t} = \left(\frac{\nu_t}{\nu_t - 2} - \xi_t^2\right) \delta_t^2 + \frac{\nu_t}{\nu_t - 2}.$$

The conditional density of r_t is[4]

$$f_{t-1}(r_t) = \sqrt{\frac{\mu_{2,t}}{\pi}} \frac{1}{\sigma_t} e^{-\frac{\delta_t^2}{2}} \nu_t^{\frac{\nu_t}{2}} (\nu_t + \varphi_t)^{-\frac{\nu_t + 1}{2}} \sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}(\nu_t + i + 1)\right)}{\Gamma\left(\frac{1}{2}\nu_t\right)} \frac{\delta_t^i}{i!} \left(\frac{2\varphi_t}{\nu_t + \varphi_t}\right)^{\frac{i}{2}},$$

where

$$\varphi_t = \frac{\mu_{2,t} (r_t - c)^2}{\sigma_t^2}.$$

One can see that in the Harvey & Siddique (1999) model, the conditional skewness is closely tied to the conditional mean specification. In particular, setting $m_t = 0$ forces $\delta_t = 0$ and, as a result, $s_t = 0$.

[3] We borrow the formula for the 3^{rd} central moment from Hogben et al. (1961).

[4] The likelihood function (2) in Harvey & Siddique (1999) does not account for non-zero mean and non-unit variance of the non-central t distribution.

C Results for individual stocks

We run the main parametric model with the 'transition' skewness specification for an arbitrary set of individual stocks belonging to the S&P100 index (and thus to the S&P500). The criterion for picking a particular stock is the availability of a long series of daily returns, comparable with the data on the index itself. This set contains the following six stocks: AEP, BA, C, IBM, T and XON. The purpose is to compare the shapes of the skewness NIC for individual stocks with that for the index that contains these stocks.

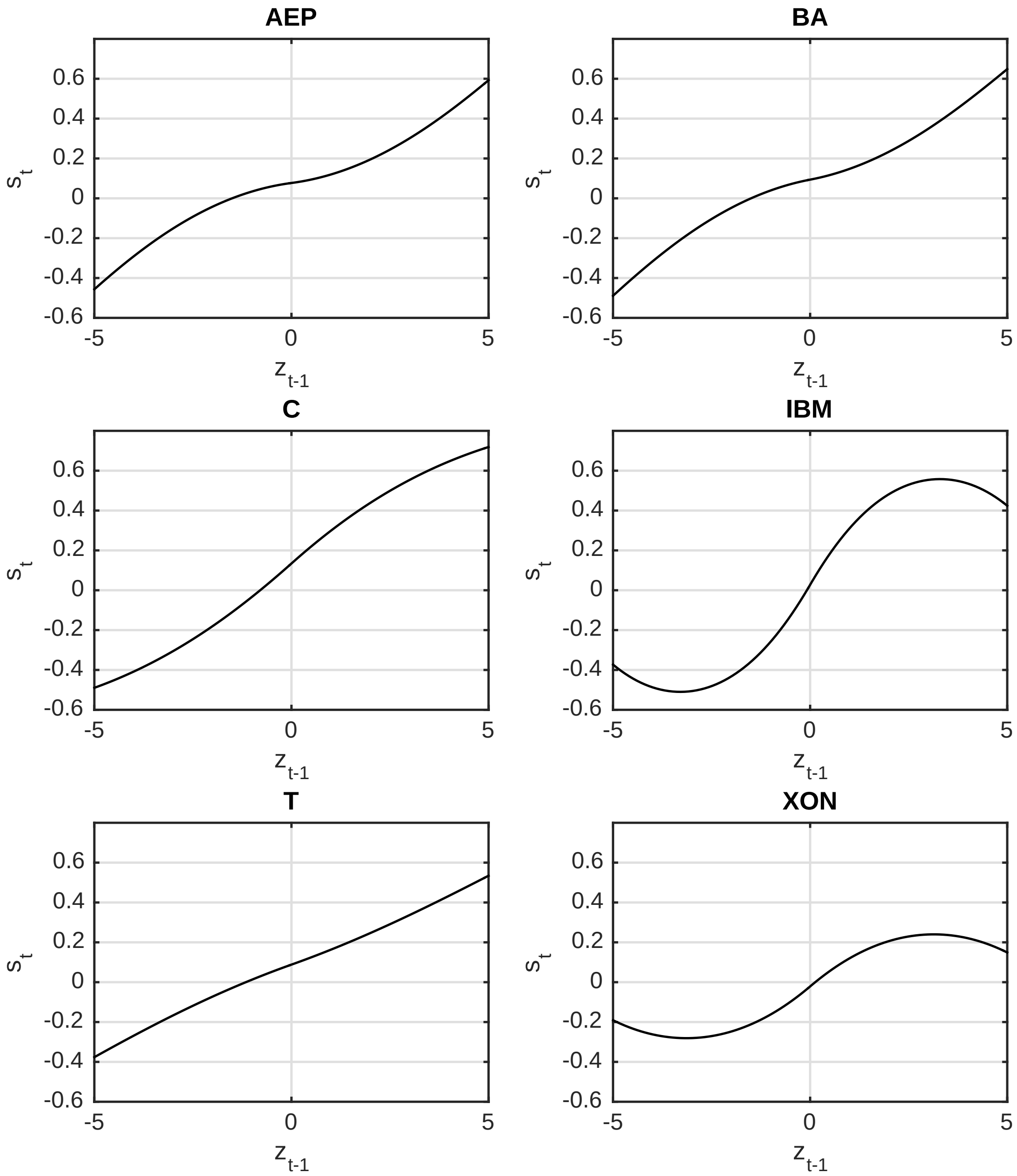
Table 2: Estimation results for individual stocks

Stock →	AEP	BA	C	IBM	T	XON
			variance equation			
β_{-1}	0.984 (0.005)	0.988 (0.004)	0.990 (0.003)	0.991 (0.003)	0.991 (0.003)	0.979 (0.006)
β_1	−0.035 (0.008)	−0.035 (0.007)	−0.035 (0.008)	−0.042 (0.008)	−0.019 (0.009)	−0.045 (0.011)
$\beta_{\lvert 1 \rvert}$	0.154 (0.024)	0.102 (0.017)	0.166 (0.025)	0.118 (0.017)	0.135 (0.024)	0.143 (0.022)
			skewness equation			
κ_0	0.041 (0.012)	0.025 (0.013)	0.042 (0.017)	0.012 (0.025)	0.045 (0.038)	−0.012 (0.023)
κ_{-1}	0.202 (0.116)	0.629 (0.131)	0.520 (0.139)	0.341 (0.103)	0.327 (0.528)	0.309 (0.148)
κ_3	0.017 (0.029)	0.027 (0.034)	0.114 (0.036)	0.238 (0.057)	0.053 (0.048)	0.133 (0.053)
ν_λ	0.641 (1.689)	0.435 (0.698)	−0.056 (0.005)	−0.152 (0.056)	0.068 (0.313)	−0.159 (0.060)
			shape parameter			
η	1.28 (0.07)	1.34 (0.04)	1.25 (0.05)	1.33 (0.04)	1.40 (0.04)	1.45 (0.04)

Notes: Robust standard errors are in parentheses. Parametric model with 'transition' specification '3' for conditional skewness is estimated.

Table 2 reports estimates for the six stocks, and Figure 2 depicts their skewness NIC. The volatility parameters show a typical pattern and do not drastically differ from those of indexes. The leverage is a bit lower, though, and the tails of the conditional distribution are heavier than those for indexes. At the same time, the skewness equation is quite different. There is moderate variability in the persistence coefficient κ_{-1}, not as big as in that for indexes. The shape of the skewness NIC, however, exhibits quite a lot of variability. On average, it is flatter than those for indexes, and shows less tendency to be non-monotonic. Interestingly, these discrepancies are not driven by a lack of big price changes in the data: for all six stocks, the proportion of standardized returns that are higher than unity in absolute value is around 27%, which is approximately equal to that of the S&P500 index; the fractions of even bigger price changes are also pretty stable.

Figure 2: Parametric skewness NIC for selected stocks from S&P100



Notes: Skewness NIC is inferred from the parametric models for conditional skewness s_t for daily AEP, BA, C, IBM, T and XON log returns depending on standardized residual z_{t-1} conditional on average $\tilde{\lambda}_{t-1}$.

D Term structure of skewness NIC

Unlike volatility, skewness does not have a scaling property: for mean zero returns r_t, while $E_t[(r_{t+1} + r_{t+2})^2] = E_t[r_{t+1}^2 + r_{t+2}^2]$, it is not the case that $E_t[(r_{t+1} + r_{t+2})^3] = E_t[r_{t+1}^3 + r_{t+2}^3]$. This makes it difficult to deduce how the skewness over long returns intervals is related to that over shorter returns subintervals. Engle (2011) shows that the unconditional skewness of the S&P500 increases in absolute value with the length of the time interval for horizons up to 100 days. Using the realized measure of skewness, Neuberger (2012) obtains similar results for horizons from one month to one year. These findings are inconsistent with the assumption of IID returns. In this section we find a similar pattern for the skewness of higher frequency returns (from 1-day to 5-day).

Table 3: Term structure of skewness NIC for S&P500 index

	S&P500				
Frequency $\rightarrow$	1	2	3	4	5
	variance equation				
β_{-1}	0.988 (0.003)	0.973 (0.005)	0.967 (0.007)	0.960 (0.006)	0.950 (0.009)
β_1	−0.074 (0.010)	−0.102 (0.013)	−0.103 (0.015)	−0.107 (0.014)	−0.120 (0.016)
$\beta_{\|1\|}$	0.117 (0.012)	0.163 (0.015)	0.173 (0.017)	0.192 (0.017)	0.210 (0.018)
	skewness equation				
κ_0	−0.050 (0.016)	−0.149 (0.042)	−0.259 (0.073)	−0.368 (0.073)	−0.559 (0.056)
κ_{-1}	0.606 (0.088)	0.394 (0.145)	0.296 (0.193)	0.081 (0.164)	−0.321 (0.081)
κ_3	0.231 (0.042)	0.103 (0.029)	0.077 (0.064)	0.107 (0.046)	0.237 (0.070)
ν_λ	−0.212 (0.021)	0.000 (0.001)	0.155 (0.341)	−0.055 (0.015)	−0.279 (0.041)
	shape parameter				
η	1.35 (0.04)	1.56 (0.05)	1.53 (0.06)	1.52 (0.06)	1.53 (0.06)
	skewness summary statistics				
mean	−0.162	−0.273	−0.411	−0.448	−0.457
st.dev.	0.250	0.118	0.103	0.097	0.151

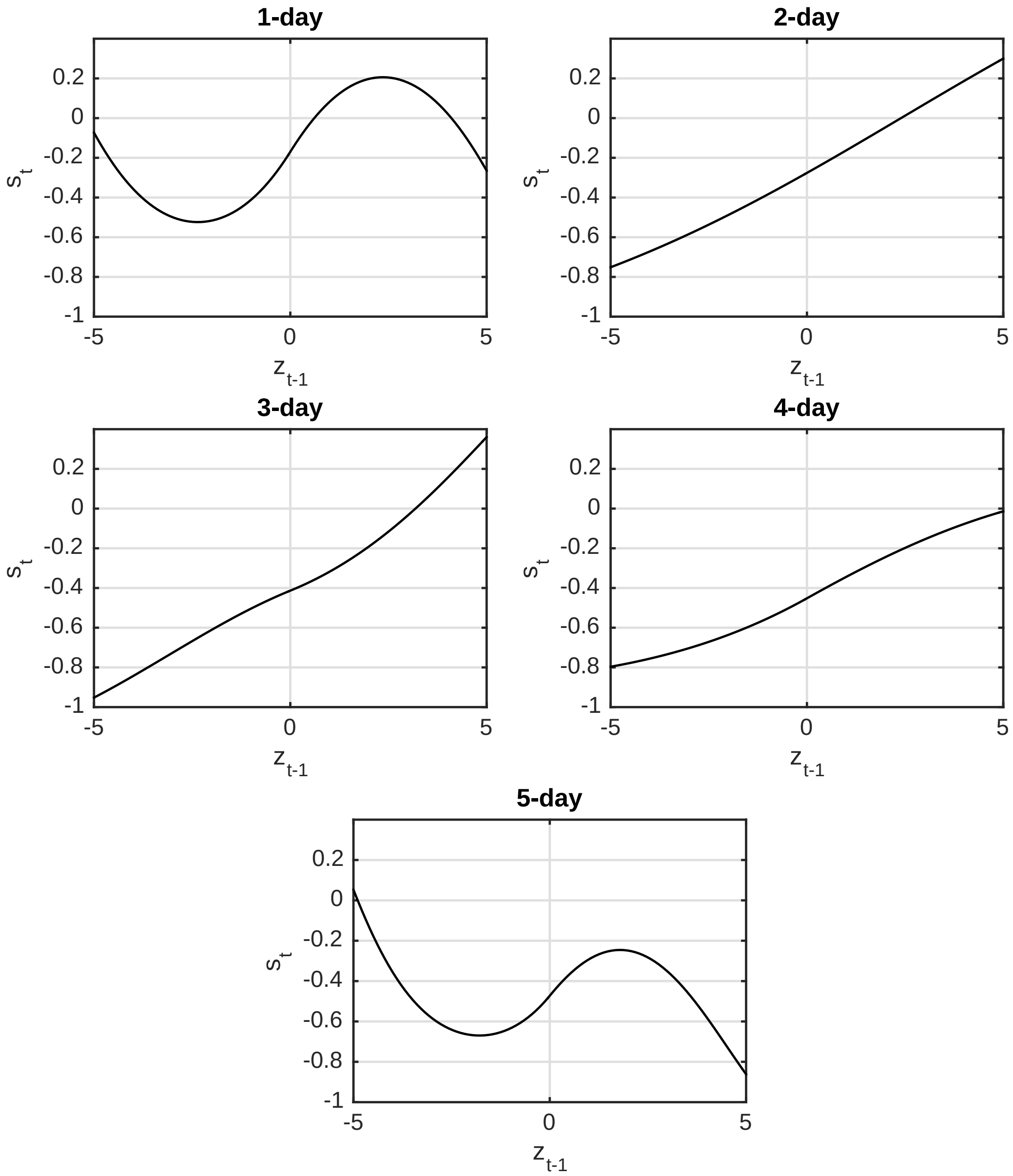
Notes: Robust standard errors are in parentheses. 'Freq' refers to data frequency: k refers to k-day returns. Parametric model with 'transition' skewness specification is estimated.

We run the main parametric model with S&P500 returns computed over several days in a row – that is, apart from the daily returns, we also compute 2-day, 3-day, 4-day and 5-day returns. In order not to lose dramatically in terms of the length of the series, we have repeated calculations with one period shift, one and two-period shifts, etc., so that in total we obtain the same number of observations as in the original series of daily returns. Another helpful feature of such a strategy is that the proportion of large and small shocks stays approximately the same as the data frequency varies.

Table 3 reports estimates for the five frequencies, and Figure 3 depicts the corresponding skewness NIC. As far as volatility is concerned, the persistence coefficient β_{-1} monotonically declines while the degrees of impact of past returns $\beta_{|1|}$ and β_1 monotonically increase as the data frequency goes down. The skewness NIC, however, shows an irregular pattern. While on average the returns become more skewed to the left and the persistence coefficient κ_{-1} goes down monotonically and quite abruptly, the shape of the skewness NIC does not exhibit non-monotonicity for 2-day, 3-day and 4-day returns. For 5-daily (i.e. weekly) returns, the skewness NIC has a 'rotated S'-shape similar to that for daily returns.

In the last panel of Table 3, we report the average implied skewness and its standard deviation. One can see that for all time horizons skewness varies substantially over time. On average, skewness is negative for all horizons and monotonically increases in absolute value with the horizon as in Engle (2011) and Neuberger (2012).

Figure 3: Parametric skewness NIC for multiperiod returns from S&P500



Notes: Skewness NIC is inferred from the parametric models for conditional skewness s_t for 1- to 5-day S&P500 log returns depending on standardized residual z_{t-1} conditional on average $\tilde{\lambda}_{t-1}$.

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